An Estimated DSGE Model of the US Economy

Rochelle M. Edge, Michael T. Kiley, and Jean-Philippe Laforte*

Preliminary and Incomplete

September 7, 2005

Abstract

This paper develops and estimates using Bayesian techiques a two-sector sticky price and wage dynamic general equilibrium model of the US economy. The model is used to generate estimates of the paths of a number of latent variables that are generally considered to be central to monetary policy formulation—specifically, the output gap and the natural rate of interest. After establishing that these measures "look sensible," the paper examines the usefulness of these measures for the conduct of monetary policy.

*Michael T. Kiley (michael.t.kiley@frb.gov) is Chief of the Macroeconomic and Quantitative Studies Section at the Board of Governors of the Federal Reserve System; Rochelle M. Edge (rochelle.m.edge@frb.gov) and Jean-Philippe Laforte (jean-philippe.laforte@frb.gov) are economists in the section. This paper represents work ongoing in the section in developing DGE models that can be useful for policy; nevertheless, any views expressed in this paper remain solely those of the authors and do not necessarily reflect those of the Board of Governors of the Federal Reserve System or it staff.

1 Introduction

This paper develops and estimates using Bayesian techniques a dynamic general equilibrium model of the US economy. The model is used to generate estimates of the paths of a number of latent variables that are generally considered to be central to monetary policy formulation—specifically, the output gap and the natural rate of interest. After establishing that these measures "look sensible," the paper examines the usefulness of these measures for the conduct of monetary policy.

The paper assumes a two-sector growth structure, with differential rates of technical progress across sectors and hence persistently divergent rates of growth across the economy's expenditure and production aggregates. This structure is necessary for the model to be consistent with recent macroeconomic phenomena that have seen large differences in the real growth rates of expenditure aggregates, along with sizeable trends in relative prices. For example, the real growth rate of gross private domestic investment over the last 25 years (1980q1 to 2004q4) has averaged around 5-1/4 percent, while real consumption growth has been averaging about 3-1/4 percent. Over this time the relative price of investment to consumption goods has declined about 1-1/2 percent per year.

The single-sector model structure, which appears to still be the most widely used set-up for DSGE models of the US economy, is unable to deliver predictions for long-term growth and relative price movements that are consistent with the above-mentioned stylized facts. Specifically, one-sector growth models, such as those that form the neoclassical-core of the models developed by Smets and Wouters (2004) and Altig, Christiano, Eichenbaum, and Linde (2004) imply that all non-stationary real variables grow at the same rate, so that over long periods of time the "great ratios" are evident in the data. Unfortunately, while this property *was* present in the 1988 dataset used by King, Plosser, Stock, and Watson (1991) in their "Stochastic Trends and Economic Fluctuation" paper it has in more recent decades failed to hold.

Single sector models also imply that there is only one price in the economy. For many reasons, we might want to avoid this assumption. First, even in the absense of divergent rates of technical progress across sectors, policymakers may be interested in knowing about more than one price index. One reason for this—relevant for the conduct of policy—is that different price indices sometimes have different degrees of price-stickiness. In the multi-sector growth set-up modeling different price indices is even more crucial. Assuming a single price index in an economy with different price levels implies the mis-measurement of all but one real variable. In an economy like the US in which inflation rates for expenditure categories (constructed using similar index number formulae) have diverged by as much as they have for as long as they have, the magnitude of this mis-measurements can become considerable, especially when looking at time periods away from the base-year.

All of this leads us to adopt a model with multiple outputs (in this case two) with different rates of technological progress. Taking account the evolution of the economy's steady-state path is, we believe, very important in model estimation, since the correct attribution of movements in macroeconomic time series to either trend movements in the data or business cycle fluctuations is vital to obtaining reliable estimates for the deep structural parameters of a model (and indeed the sequence of shocks underlying the data). The precise two-sector structure that we employ is based on the observation by Whelan (2001) that while real growth rates differ considerably across expenditure categories, nominal growth rates are more similar. For example, nominal consumption growth has averaged about 6-3/4 percent over the last quarter century while nominal gross private domestic investment growth has averaged around 7 percent. This implies that while the "great ratios" in *real* terms no longer hold in the data their *nominal* counterparts do. The type of two-sector model that delivers this long-term prediciton is one in which production in each sector of the economy is characterized by a Cobb-Douglas production function in labor, capital, and technology, where the technology processes differs between the economy's two sectors and have divergent trend growth rates. This model is the core-neoclassical growth model that underlies the model with real and nominal rigidities outlined in section 2 through 4 of the paper. When this model is estimated in section 5, we allow the Kalman filter to perform the stochastic detrending of the model and to estimate the sector's steady-state growth rates. The model's properties are presented in section 6 and preliminary policy analysis is conducted in section 7.

2 The Production and Preference Technologies

In this section we present the production and preference technologies for our two-sector growth model. The long-run evolution of the economy is determined by differential rates of stochastic growth in the two sectors of the economy, while its short-run dynamics are influenced by various forms of adjustment costs. Adjustment costs to real aggregate variables are captured by the economy's preference and production technologies presented in this section. Adjustment costs to real sectoral variables and nominal variables are captured in the decentralization of the model presented in the following section.

2.1 The Production Technology

Two distinct final goods are produced in our model economy: consumption goods (denoted $Y_t^{f,c}$) and capital goods (denoted $Y_t^{f,k}$). These final goods are produced by aggregating—according to a Dixit-Stiglitz technology—an infinite number of differentiated inputs. Specifically, final goods production is represented by the function

$$Y_t^{f,s} = \left(\int_0^1 Y_t^{f,s}(j)^{\frac{\Theta_t^{y,s}-1}{\Theta_t^{y,s}}} dj \right)^{\frac{\Theta_t^{y,s}}{\Theta_t^{y,s}-1}}, \qquad s = c, k, \tag{1}$$

where the variable $Y_t^{f,s}(j)$ denotes the quantity of the jth input (obtained from the intermediate goods sector) used to produce final output $s = c$ or $s = k$ while $\Theta_t^{y,s}$ is the stochastic elasticity of substitution between the differentiated intermediate goods inputs used in the production of the consumption or capital goods sectors. Letting $\theta_t^{y,s} \equiv \ln \Theta_t^{y,s} - \ln \Theta_*^{y,s}$ denote the log-deviation of $\Theta_t^{y,s}$ from its steady-state value of $\Theta_*^{y,s}$, we assume that

$$\theta_t^{y,s} = \rho^{\theta,y,s} \theta_{t-1}^{y,s} + \epsilon_t^{\theta,y,s} \tag{2}$$

where $\epsilon_t^{\theta,y,s}$ is an i.i.d. shock process, and $\rho^{\theta,y,s}$ represents the persistence of $\Theta_t^{y,s}$ away from steady-state following a shock to equation (2).

The jth differentiated intermediate good in sector s (which is used as an input in equation 1) is produced by combining each variety of the economy's differentiated labor inputs $\{L_t^{y,s}(i,j)\}_{i=0}^1$ with the sector's specific capital stock $K_t^s(j)$. A Dixit-Stiglitz aggregator characterizes the way in which differentiated labor inputs are combined to yield a composite bundle of labor, denoted $L_t^{y,s}(j)$. A Cobb-Douglas production function then characterizes how this composite bundle of labor is used with capital to produce—given the current level of multifactor productivity MFP_t^s in the sector s—the intermediate good $Y_t^{m,s}(j)$. The production of intermediate good j is represented by the function:

$$Y_t^{m,s}(j) = (K_t^s(j))^{\alpha} \left(\underbrace{A_t^m Z_t^m A_t^s Z_t^s}_{MFP_t^s} L_t^{y,s}(j) \right)^{1-\alpha} \text{ where } L_t^{y,s}(j) = \left(\int_0^1 L_t^{y,s}(i,j)^{\frac{\Theta_t^{l,s}-1}{\Theta_t^{l,s}}} di \right)^{\frac{\Theta_t^{l,s}}{\Theta_t^{l,s}-1}} \quad s = c,k. \tag{3}$$

The parameter α in equation (3) is the elasticity of output with respect to capital while $\Theta_t^{l,s}$ denotes the stochastic elasticity of substitution between the differentiated labor inputs. Letting $\theta_t^{l,s} \equiv \ln \Theta_t^{l,s} - \ln \Theta_*^{l,s}$ denote the log-deviation of $\Theta_t^{l,s}$ from its steady-state value of Θ_*^l, we assume that

$$\theta_t^{l,s} = \rho^{\theta,l,s} \theta_{t-1}^{l,s} + \epsilon_t^{\theta,l,s} \tag{4}$$

where $\epsilon_t^{\theta,l,s}$ is an i.i.d. shock process, and $\rho^{\theta,l,s}$ represents the persistence of $\Theta_t^{l,s}$ away from steady-state following a shock to equation (4).

The level of technology in sector s has four components. The A_t^m and Z_t^m components represent economy-wide technology shocks, while the A_t^s and Z_t^s terms (for $s = c,k$) represent technology shocks that are specific to either the consumption or capital goods sectors. The A_t technology terms represent shocks that exhibit only transitory movements away from their steady-state unit mean, while the Z_t technology terms represent shocks that exhibit permanent movements in their levels. Specifically, letting $a_t^s \equiv \ln A_t^s$ denote the log-deviation of A_t^s from its steady-state value of unity, we assume that

$$a_t^s = \rho^{a,s} a_{t-1}^s + \epsilon_t^{a,s}, \;\; s = c,k,m \tag{5}$$

where $\epsilon_t^{a,s}$ is an i.i.d. shock process, and $\rho^{a,s}$ represents the persistence of A_t^s away from steady-state following a shock to equation (5). The stochastic process Z_t^s evolves according to

$$\ln Z_t^s - \ln Z_{t-1}^s = \ln \Gamma_t^{z,s} = \ln \left(\Gamma_*^{z,s} \cdot \exp[\gamma_t^{z,s}] \right) = \ln \Gamma_*^{z,s} + \gamma_t^{z,s}, \;\; s = c,k,m \tag{6}$$

where $\Gamma_*^{z,s}$ and $\gamma_t^{z,s}$ are the steady-state and stochastic components of $\Gamma_t^{z,s}$. The stochastic component $\gamma_t^{z,s}$ is assumed to evolve according to

$$\gamma_t^{z,s} = \rho^{z,s,} \gamma_{t-1}^{z,s} + \epsilon_t^{z,s}. \tag{7}$$

where $\epsilon_t^{z,s}$ is an i.i.d shock process, and $\rho^{z,s}$ represents the persistence of $\gamma_t^{z,s}$ to a shock. In line with historical experience, we assume a more rapid rate of technological progress in capital goods production by calibrating the steady-state growth rate of the non-stationary component of technology in the capital goods sector above that in the consumption goods sector. That is, $\Gamma_*^{z,k} > \Gamma_*^{z,c} (= 1)$, where an asterisk on a variable denotes its steady-state value.

2.2 Capital Stock Evolution

Purchases of the economy's capital good can be transformed into capital that can then be used in the production the economy's two goods. The kth capital owner's beginning of period $t+1$ capital stock $K_{t+1}(k)$ is equal to the previous periods undepreciated capital stock $(1-\delta)K_t(k)$, augmented by new capital installed in the previous period $I_t(k)$. We assume that not all investment expenditure results in productive capital, since some fraction is absorbed by adjustment costs in the process of installation. Specifically, the evolution of the economy's capital stock is given by

$$K_{t+1}(k) = (1-\delta)K_t(k) + I_t(k) - \frac{\chi^i}{2}\left(\frac{I_t(k) - \eta^i I_{t-1}\Gamma_*^k - (1-\eta^i)\widetilde{I}_* Z_t^m Z_t^k}{K_t}\right)^2 K_t, \tag{8}$$

where $\widetilde{I}_*$ denotes the value of steady-state investment spending normalized by the permanent component of technology so as to be constant in the steady state. Note that investment adjustment costs are zero when $\frac{I_t}{Z_t^m Z_t^k} = \frac{I_{t-1}}{Z_{t-1}^m Z_{t-1}^k} = \widetilde{I}_*$ but rise to above zero, at an increasing rate, as investment growth moves further away from this. The costs for altering investment depend on both the level of (growth-adjusted) investment spending from the preceding period as well as the steady-state level of investment spending. The parameter χ^i governs how quickly these costs increase away from the steady-state.

2.3 Preferences

The ith household derives utility from its purchases of the consumption good $C_t(i)$ and from the use of its leisure time, which is equal to what remains of its time endowment after $L_t^c(i) + L_t^k(i)$ hours of labor are used up through working. The preferences of household i over consumption and leisure are separable, with household i's consumption habit stock (assumed to equal a factor h multiplied by its consumption last period $C_{t-1}(i)$) influencing the utility it derives from current consumption. Specifically, the preferences of household i are represented by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\Xi_t^b \ln(C_t(i) - hC_{t-1}(i)) - \varsigma \Xi_t^l \frac{(L_t^{u,c}(i) + L_t^{u,k}(i))^{1+\nu}}{1+\nu} \right]. \tag{9}$$

The parameter β is the household's discount factor, ν denotes its labor supply elasticity, while ς is a scale parameter. The stationary, unit-mean, stochastic variables Ξ_t^b and Ξ_t^l represent aggregate shocks to the household's utility of consumption and disutility of labor. Letting $\xi_t^x \equiv \ln \Xi_t^x - \ln \Xi_*^x$ denote the log-deviation of Ξ_t^x from its steady-state value of Ξ_*^x, we assume that

$$\xi_t^x = \rho^{\xi,x} \xi_{t-1}^x + \epsilon_t^{\xi,x}, \qquad \upsilon = b, l. \tag{10}$$

The variable $\epsilon_t^{\xi,x}$ is an i.i.d. shock process, and $\rho^{\xi,x}$ represents the persistence of Ξ_t^x away from steady-state following a shock to equation (10).

3 The Decentralized Economy

We assume the following decentralization of the economy. There is one representative, perfectly competitive firm in each of the two final-goods producing sectors, which purchases intermediate inputs from the continuum of intermediate goods producers. The intermediate goods producers, in turn, rent capital from a perfectly competitive representative capital owner, and differentiated types of labor from households. The capital owner purchase the capital good from the (final) capital-goods producing firm, and households purchase the consumption good from the (final) consumption-goods producing firm. Because both intermediate goods producers and households are monopolistic competitors, they also set the prices or wages at which they supply their respective products or labor services.

3.1 Consumption and Capital Final Goods Producers

The competitive firm in the consumption good sector owns the production technology described in equation (1) for $s = c$, while the competitive firm in the capital goods sector owns the same technology for $s = k$.

The final-good producing firm in sector s takes as given the prices $\{P_t^s(j)\}_{j=0}^1$ set by each intermediate good producing firm for its differentiated output, and choose $\{Y_t^{m,s}(j)\}_{j=0}^1$ to minimize its production costs subject to the aggregator function. Specifically, the final-good producer in sector s solves the cost-minimization problem of:

$$\min_{\{Y_t^{m,s}(j)\}_{j=0}^1} \int_0^1 P_t^s(j) Y_t^{m,s}(j) dj \text{ subject to } \left(\int_0^1 \left(Y_t^s(j)\right)^{\frac{\Theta_t^{y,s}-1}{\Theta_t^{y,s}}} dj \right)^{\frac{\Theta_t^{y,s}}{\Theta_t^{y,s}-1}} \geq Y_t^s, \text{ for } s = c, k. \tag{11}$$

The cost-minimization problems solved by firms in the economy's consumption and capital goods producing sectors imply demand functions for each intermediate good that are given by $Y_t^s(j) = \left(P_t^s(j)/P_t^s\right)^{-\Theta_t^{y,s}} Y_t^s$. The variable P_t^s, which denotes the aggregate price level in the final goods sector s, is defined by $P_t^s = \left(\int_0^1 (P_t^s(j))^{1-\Theta_t^{y,s}} dj\right)^{\frac{1}{1-\Theta_t^{y,s}}}$.

3.2 Consumption and Capital Intermediate Goods Producers

Each intermediate-good producing firm $j \in [0,1]$ and $s = c, k$ owns the production technology described in equation (3). In describing the intermediate good producing firm's problem it is convenient to split it into three separate stages.

In the first stage of the problem firm j in sector s, taking as given the wages $\{W_t^s(i)\}_{i=0}^1$ set by each household for its variety of labor supplied to sector s, chooses $\{L_t^{y,s}(i,j)\}_{i=0}^1$ to minimize the cost of attaining the aggregate labor bundle $L_t^{y,s}(j)$ that it will ultimately need for production. Specifically, the intermediate firm j solves:

$$\min_{\left\{L_t^{y,s}(i,j)\right\}_{i=0}^1} \int_0^1 W_t^s(i) L_t^{y,s}(i,j) di \text{ subject to } \left(\int_0^1 \left(L_t^{y,s}(i,j)\right)^{\frac{\Theta_t^{l,s}-1}{\Theta_t^{l,s}}} di\right)^{\frac{\Theta_t^{l,s}}{\Theta_t^{l,s}-1}} \geq L_t^s(j), \text{ for } s = c, k. \tag{12}$$

This cost-minimization problem undertaken by each intermediate good producing firm implies that the demand in sector s for type i labor is $L_t^{y,s}(i) = \int_0^1 L_t^{y,s}(i,j)dj = \left(W_t^s(i)/W_t^s\right)^{-\Theta_t^{l,s}}$ $\times \int_0^1 L_t^{u,s}(j)dj$ where W_t^s denotes the aggregate wage for labor supplied to sector s, defined by $W_t^s = \left(\int_0^1 (W_t^s(x))^{1-\Theta_t^{l,s}} dx\right)^{\frac{1}{1-\Theta_t^{l,s}}}$.

In the second stage of the problem firm j in sector s, taking as given the aggregate sector s wage W_t^s and the sector s rental rate on capital $R_t^{k,s}$, chooses aggregate labor $L_t^{y,s}(j)$ and capital $K_t^s(j)$ to minimize the costs of attaining its desired level of output $Y_t^s(j)$. Specifically, firm j in sector s solves

$$\min_{\left\{L_t^{y,s}(j), K_t^s(j)\right\}} W_t^s L_t^{y,s}(j) + R_t^{k,s} K_t^s(j) \text{ s.t. } \left(A_t^m Z_t^m A_t^s Z_t^s L_t^{y,s}(j)\right)^{1-\alpha} \left(K_t^s(j)\right)^{\alpha} \geq Y_t^s(j), \text{ for } s = c, k. \tag{13}$$

Since each intermediate goods firm produces its own differentiated variety of output $Y_t^{m,s}(j)$, it is able to set its price $P_t^s(j)$. It does this taking into account the demand schedule for its output that it faces from the final-goods sector s, $Y_t^s(j) = \left(P_t^s(j)/P_t^s\right)^{-\Theta_t^{y,s}} Y_t^s$, as well as the adjustment costs that it faces in altering its price. These adjustment costs are introduced to the model through the assumption that the act of altering prices absorbs some of the intermediate-goods producing firm's output $Y_t^{m,s}(j)$, so leaving a somewhat diminished amount, $Y_t^{f,s}(j)$, available to be sold as an input into final good production. Specifically, we assume that:

$$Y_t^{f,s}(j) = Y_t^{m,s}(j) - \frac{100 \cdot \chi^{p,s}}{2} \left(\frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^{p,s} \Pi_{t-1}^{p,s} - (1-\eta^{p,s}) \Pi_*^{p,s} \right)^2 Y_t^{m,s}, \text{ for } s = c, k. \tag{14}$$

where the parameter $\eta^{p,s}$ reflects the importance of lagged inflation relative to steady-state inflation in determining adjustment costs and $\Pi_*^{p,s}$ denotes the steady-state rate of sector s price inflation. The parameter $\chi^{p,s}$ scales linearly the magnitude of the adjustment cost term; in a flexible price model $\chi^{p,s}$ wouble be equal to zero. In what follows we consider the most general case of the firm's profit-maximization problem, that is, the one in which there are positive price adjustment costs; the first-order conditions implied from a model with sticky prices can be trivially converted to those of the model with flexible prices by simply setting the price adjustment cost parameter equal to zero.

In the profit-maximizing part of its problem, the intermediate-good producing firm j, taking as given the marginal cost $MC_t^s(j)$ for producing $Y_t^s(j)$, the aggregate sector s price level P_t^s, and final output Y_t^s by sector s, chooses its price $P_t^s(j)$ to maximize the present discounted value of its profits subject to the demand curve it faces for its differentiated output and the costs it faces in adjusting its price (equation **??**). Since the intermediate firms are ultimately owned by the economy's households, intermediate producers act on their behalf when making their profit-maximizing price-setting decisions. For this reason the intermediate firms value their revenues and costs across time exactly as the household would value them. Consequently, the discount factor that is relevant when comparing nominal revenues and costs in period t with those in period $t+j$ is $\beta^j \frac{\Lambda_{t+j}^c / P_{t+j}^c}{\Lambda_t^c / P_t^c}$, where $\Lambda_t^c = \int_0^1 \Lambda_t^c(i) di$. The variable $\Lambda_t^c(i)$ denotes the household i's marginal utility of consumption in period t, which implies that Λ_t^c is the average marginal utility of consumption across households. Put formally, the sector s intermediate-good producing firm's profit-maximization problem is:

$$\max_{\{P_t^s(j),Y_t^{m,s}(j),Y_t^{f,s}(j)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^c}{P_t^c} \left\{ \left(1+\sigma^{p,s}\right) P_t^s(j) Y_t^{f,s}(j) - MC_t^s(j) Y_t^{m,s}(j) \right\}$$

subject to

$$Y_\tau^{m,s}(j) = \left(\frac{P_\tau^s(j)}{P_\tau^s}\right)^{-\Theta_\tau^{y,s}} Y_\tau^{m,s} \text{ and}$$

$$Y_\tau^{f,s}(j) = Y_\tau^{m,s}(j) - \frac{100 \cdot \chi^{p,s}}{2} \left(\frac{P_\tau^s(j)}{P_{\tau-1}^s(j)} - \eta^{p,s} \Pi_{\tau-1}^{p,s} - (1-\eta^{p,s}) \Pi_*^{p,s} \right)^2 Y_\tau^{m,s}$$

$$\text{for } \tau = 0, 1, ..., \infty, \text{ and } s = c, k. \tag{15}$$

The parameter $\sigma^{p,s} = (\Theta_*^{p,s} - 1)^{-1}$ is a subsidy to production that is set to ensure that the economy's level of steady-state output is Pareto optimal.

3.3 Capital Owners

Capital owners possess the technology described in equation (8) for transforming capital goods, purchased from capital final-goods producing firm, into a capital stock that can be used in the production of the economy's two diferentiated intermediate goods.

We assume that capital owners face a cost in moving capital between the two intermediate goods producing sectors of the economy (but do not encounter any cost in moving capital between firms within the same sector). Specifically, the relationship between the aggregate capital stock $K_t(k)$ and the capital stocks used in the consumption and capital intermediate goods productiing sectors, that is $K_t^c(k)$ and $K_t^k(k)$, is given by:

$$K_t^c(k) + K_t^k(k) = K_t(k) - \frac{100 \cdot \chi^k}{2} \left(\frac{K_t^c(k)}{K_t^k(k)} - \eta^k \frac{K_{t-1}^c}{K_{t-1}^k} - (1-\eta^k) \frac{\widetilde{K}_*^c}{\widetilde{K}_*^k} \right)^2 \frac{K_t^k}{K_t^c} \cdot K_t. \tag{16}$$

where the parameter η^k reflects the importance of lagged composition of capital supply relative to the steady-state composition, $\widetilde{K}_*^c/\widetilde{K}_*^k$, in determining adjustment costs. The parameter χ^k scales linearly the magnitude of the adjustment cost term.

The representative competitive capital owner, taking as given the rental rate on capital in the economy's two sectors, $R_t^{k,c}$ and $R_t^{k,k}$, the price of capital goods P_t^k, and the stochastic discount factor $\beta^j \frac{\Lambda_{t+j}^c/P_{t+j}^c}{\Lambda_t^c/P_t^c}$, chooses investment, $I_t(k)$ and the capital stocks it supplies to the economy's two sectors $K_t^c(k)$ and $K_t^k(k)$, to maximize the present discounted value of profits subject to the law of motion governing the evolution of capital (equation 8) and given the costs implied by moving capital between the sectors (equation 16).[1] Specifically, the capital owner solves:

$$\max_{\{I_t(k),K_{t+1}(k),K_t^c(k),K_t^k(k)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^c}{P_t^c} \left\{ R_t^{k,c} K_t^c(k) + R_t^{k,k} K_t^k(k) - P_t^k I_t(k) \right\}$$

subject to

$$K_{\tau+1}(k) = (1-\delta)K_\tau(k) + I_\tau(k) - \frac{100\cdot\chi^i}{2}\left(\frac{I_\tau(k) - \eta^i I_{\tau-1}(k)\Gamma_t^{y,k} - (1-\eta^i)\widetilde{I}_* Z_\tau^m Z_\tau^k}{K_\tau}\right)^2 K_\tau$$

$$K_\tau^c(k) + K_\tau^k(k) = K_\tau(k) - \frac{100\cdot\chi^k}{2}\left(\frac{K_\tau^c(k)}{K_\tau^k(k)} - \eta^k\frac{K_{\tau-1}^c}{K_{\tau-1}^k} - (1-\eta^k)\frac{\widetilde{K}_*^c}{\widetilde{K}_*^k}\right)^2 \frac{K_\tau^k}{K_\tau^c}\cdot K_\tau.$$

for $\tau = 0, 1, ..., \infty$. (17)

3.4 Households

Household's utility, which is defined over consumption and leisure, is described by equation (9).

Since each household supplies its own differentiated variety of labor to each sector, $L_t^{u,s}(i)$, it is able to set its wage $W_t^s(i)$. It does this taking into account the demand schedule for its labor that it faces from intermediate goods sector s and the adjustment costs that it encounters in altering its wage and the composition of its labor. Analogous to to the constraint faced by intermediate goods producers, wage-setting adjustment costs are introduced to the model through the assumption that the act of altering wages absorbs some of the household's time endowment resources, which implies that not all of the hours that the household devotes to working $L_t^{u,s}$ results in productive wage-earning hours $L_t^{y,s}$. In addition we assume that redircting labor from one sector to the other is also diverts time away from leisure that does not showing up as productive wage-earning hours. This latter cost is split between the two types of labor that the household supplies to the market. Together these adjustment costs imply that

$$\begin{aligned} L_t^{y,s}(i) = L_t^{u,s}(i) &- \frac{100\cdot\chi^{w,s}}{2}\left(\frac{W_t^s(j)}{W_{t-1}^s(j)} - \eta^{w,s}\Pi_{t-1}^{w,s} - (1-\eta^{p,s})\Pi_*^w\right)^2 L_t^{u,s} \\ &- \frac{L_*^{u,s}}{L_*^{u,c}+L_*^{u,k}}\cdot\frac{10\cdot\chi^l}{2}\left(\frac{L_t^{u,c}(i)}{L_t^{u,k}(i)} - \eta^l\frac{L_{t-1}^{u,c}}{L_{t-1}^{u,k}} - (1-\eta^l)\frac{L_*^{u,c}}{L_*^{u,k}}\right)^2 \frac{L_t^{u,k}}{L_t^{u,c}},\ \text{for } s = c, k. \end{aligned}$$

(18)

[1] The economy's capital stock is also ultimately owned by the households, so that the relevant discount factor in comparing nominal earnings and expenditures in period t with those in period $t+j$ is $\beta^j \frac{\Lambda_{t+j}^c/P_{t+j}^c}{\Lambda_t^c/P_t^c}$.

The parameter $\eta^{w,s}$ reflects the importance of lagged wage inflation relative to steady-state inflation in determining adjustment costs and Π_*^w denotes the steady-state rate of wage inflation (which is equal across sectors). The parameter $\chi^{w,s}$ scales linearly the magnitude of the adjustment cost term; in a flexible price model $\chi^{w,s}$ wouble be equal to zero. The parameter η^l reflects the importance of lagged composition of labor supply relative to the steady-state composition, $L_*^{u,c}/L_*^{u,k}$, in determining adjustment costs. The parameter χ^l scales linearly the magnitude of the adjustment cost term.

The household's budget constraint is given by

$$\mathcal{E}_t\left[R_t^{-1}B_{t+1}(i)\right] = B_t(i) + \sum_{s=c,k}(1+\sigma^{w,s})W_t^s(i)L_t^{y,s}(i) + Profits_t(i) - P_t^c C_t(i) \tag{19}$$

where the variable $B_t(i)$ is the state-contingent value, in terms of the numeraire, of household i's asset holdings at the beginning of period t. We assume that there exists a riskfree one-period bond, which pays one unit of the numeraire in each state, and denote its yield—that is, the gross nominal interest rate between periods t and $t+1$—by $R_t \equiv \left(\mathcal{E}_t\beta\frac{\Lambda_{t+1}^c/P_{t+1}^c}{\Lambda_t^c/P_t^c}\right)^{-1}$. Profits are those repatriated from capital owner and intermediate good producing firms who, as already noted, are ultimately owned by households. The parameter $\sigma^{w,s} = (\Theta_*^{w,s} - 1)^{-1}$ is a subsidy to labor that is set to ensure that the economy's level of steady-state labor (and consequently output) is Pareto optimal.

The household, taking as given the expected path of the gross nominal interest rate R_t, the consumption good price level P_t^c, the aggregate wage rate in each sector W_t^s, profits income, and the initial bond stock $B_0(i)$, chooses its consumption $C_t(i)$ and its wage in each sector $W_t^s(i)$ to maximize its utility subject to its budget constraint and the demand curve it faces for its differentiated labor. Specifically, the household solves:

$$\max_{\left\{C_t(i),\{W_t^s(i),L_t^{u,s}(i),L_t^{y,s}(i)\}_{s=c,k},B_{t+1}(i)\right\}_{t=0}^{\infty}} \mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\Xi_t^b \ln(C_t(i)-hC_{t-1})-\varsigma\Xi_t^l\frac{(L_t^{u,c}(i)+L_t^{u,k}(i))^{1+\nu}}{1+\nu}\right\}$$

subject to

$$\mathcal{E}_t\left[R_\tau^{-1}B_{\tau+1}(i)\right]=B_\tau(i)+\sum_{s=c,k}(1+\sigma^{w,s})W_\tau^s(i)L_\tau^{y,s}(i)+\mathit{Profits}_\tau(i)-P_\tau^c C_\tau(i)$$

$$L_\tau^{y,s}(i)=L_\tau^{u,s}(i)-\frac{100\cdot\chi^{w,s}}{2}\left(\frac{W_\tau^s(j)}{W_{\tau-1}^s(j)}-\eta^{w,s}\Pi_{\tau-1}^{w,s}-(1-\eta^{p,s})\Pi_*^w\right)^2 L_\tau^{u,s}$$
$$-\frac{L_*^{u,s}}{L_*^{u,c}+L_*^{u,k}}\cdot\frac{10\cdot\chi^l}{2}\left(\frac{L_\tau^{u,c}(i)}{L_\tau^{u,k}(i)}-\eta^l\frac{L_{\tau-1}^{u,c}}{L_{\tau-1}^{u,k}}-(1-\eta^l)\frac{L_*^{u,c}}{L_*^{u,k}}\right)^2\frac{L_\tau^{u,k}}{L_\tau^{u,c}},\ \text{for } s=c,k.$$

$$L_\tau^{y,c}(i)=\left(\frac{W_\tau^c(i)}{W_\tau^c}\right)^{-\Theta_\tau^{l,c}}L_\tau^{y,c},\ \text{and}\ L_\tau^{y,k}(i)=\left(\frac{W_\tau^k(i)}{W_\tau^k}\right)^{-\Theta_\tau^{l,k}}L_\tau^{y,k},\quad \text{for } \tau=0,1,...,\infty. \tag{20}$$

3.5 Goods and Factor Market Clearing

We note the following goods and factor market clearing conditions. The market clearing conditions for labor and capital supplied and demanded in sector s are given by

$$L_t^{y,s}(i)=\int_0^1 L_t^{y,s}(i,j)dj\ \text{ and }\ \int_0^1 K_t^s(k)dk=\int_0^1 K_t^s(j)dj\ \text{ for all } i\in[0,1] \text{ and for } s=c,k. \tag{21}$$

The market clearing conditions for final consumption goods output and consumption expenditure and final capital goods output and investment expenditure are is given by

$$Y_t^{f,c}=\int_0^1 C_t(i)di\ \text{ and }\ Y_t^{f,k}=\int_0^1 I_t(k)dk. \tag{22}$$

3.6 Identities

The model also consists of the following identities:

$$W_t^s(i)=\Pi_t^{w,s}(i)W_{t-1}^s(i)\ \text{ and }\ W_t^s=\Pi_t^{w,s}W_{t-1}^s\ \text{ for all } i\in[0,1] \text{ and for } s=c,k, \text{ and} \tag{23}$$

$$P_t^s(i)=\Pi_t^{p,s}(i)P_{t-1}^s(i)\ \text{ and }\ P_t^s=\Pi_t^{p,s}P_{t-1}^s\ \text{ for all } i\in[0,1] \text{ and for } s=c,k. \tag{24}$$

3.7 Aggregate Output and Aggregate Price Inflation

As will be discussed shortly the central bank sets monetary policy in accordance with an interest rate rule that responds to GDP output growth and GDP inflation—variables that have not yet been defined in our model. Multi-sector models do not possess—or indeed necessarily require—any aggregate output or aggregate price concept but to the extent that monetary policy is interested in these variables in setting interest rates it is necessary for us to construct such concepts. We choose construct our aggregate activity and price inflation variables in the same way that the Bureau of Economic Analysis (BEA) does in producing the National Income and Product Accounts (NIPA). Specifically, real GDP growth, $H_t^{y,gdp}$ is a chain-weighted aggregate of output growth in the consumption and investment goods sectors—that is, $Y_t^{y,c}/Y_{t-1}^{y,c}$ and $Y_t^{y,k}/Y_{t-1}^{y,k}$—and the growth of autonomous output—denoted $H^{y,gf}$ (which is itself the chain-weighted sum of government and foreign demand). GDP price inflation, $\Pi_t^{p,gdp}$, is a chain-weighted aggregate of consumption and capital goods price inflation—that is, $\Pi_t^{p,c} = P_t^c/P_{t-1}^c$ and $\Pi_t^{p,k} = P_t^k/P_{t-1}^k$—and the rate of increase of the autonomous output price deflator—$\Pi_t^{p,gf}$. The precise formulas for these aggregate variables are given by

$$
\begin{aligned}
H_t^{y,gdp} = \left(Y_t^c/Y_{t-1}^c\right)^{\frac{1}{2}\cdot\frac{P_t^cY_t^c}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^cY_{t-1}^c}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}} \\
\times\left(Y_t^k/Y_{t-1}^k\right)^{\frac{1}{2}\cdot\frac{P_t^kY_t^k}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^kY_{t-1}^k}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}} \\
\times\left(H_t^{y,gf}\right)^{\frac{1}{2}\cdot\frac{P_t^{gf}Y_t^{gf}}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^{gf}Y_{t-1}^{gf}}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}} \qquad (25)
\end{aligned}
$$

and

$$
\begin{aligned}
\Pi_t^{p,gdp} = \left(\Pi_t^{p,c}\right)^{\frac{1}{2}\cdot\frac{P_t^cY_t^c}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^cY_{t-1}^c}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}} \\
\times\left(\Pi_t^{p,k}\right)^{\frac{1}{2}\cdot\frac{P_t^kY_t^k}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^kY_{t-1}^k}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}} \\
\times\left(\Pi_t^{p,gf}\right)^{\frac{1}{2}\cdot\frac{P_t^{gf}Y_t^{gf}}{P_t^cY_t^c+P_t^kY_t^k+P_t^{gf}Y_t^{gf}}+\frac{1}{2}\cdot\frac{P_{t-1}^{gf}Y_{t-1}^{gf}}{P_{t-1}^cY_{t-1}^c+P_{t-1}^kY_{t-1}^k+P_{t-1}^{gf}Y_{t-1}^{gf}}}. \qquad (26)
\end{aligned}
$$

We normalized the autonomous output price deflator to that of the consumption good sector. The growth rate of autonomous output H_t^{gf} is exogenous to our model and assumed to follow AR(1) process. Specifically, letting $h_t^{y,gf} = \ln H_t^{y,gf} - \ln H_*^{y,gf}$, we allow

$$h_t^{gf} = \rho^{h,gf}\cdot h_{t-1}^{gf} + \epsilon_t^{h,gf}.$$

3.8 Monetary Authority

The central bank sets monetary policy in accordance with an Taylor-type interest-rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\phi^r} \left(\bar{R}_t\right)^{1-\phi^r} \exp\left[\epsilon_t^r\right], \tag{27}$$

where the parameter ϕ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate $\bar{R}_t$ is given by:

$$\bar{R}_t = \left(\Pi_t^{p,gdp}/\Pi_*^{p,gdp}\right)^{\phi^{\pi,gdp}} \left(\Delta\Pi_t^{p,gdp}\right)^{\phi^{\Delta\pi,gdp}} \left(H_t^{y,gdp}/H_*^{y,gdp}\right)^{\phi^{h,gdp}} \left(\Delta H_t^{y,gdp}\right)^{\phi^{\Delta h,gdp}} R_*. \tag{28}$$

where R_* denotes the economy's steady-state nominal interest rate (which is equal to $(1/\beta)\Pi_*^{p,c}\Gamma_*^{z,m}(\Gamma_*^{z,k})^{\alpha}(\Gamma_*^{z,c})^{1-\alpha}$) and $\phi^{\pi,gdp}$, $\phi^{\Delta\pi,gdp}$, $\phi^{h,gdp}$, and $\phi^{\Delta h,gdp}$ denote the weights in the feedback rule.

3.9 Equilibrium

Before characterizing equilibrium in this model, we define one additional variable, the price of installed capital $Q_t^k(k)$. This variable is equal to the lagrange multiplier on the capital evolution equation that would be implied by the kth capital owner's profit-maximization problem (equation 17).

Equilibrium in our model is an allocation:

$$\Big\{H_t^{y,gdp}, Y_t^{f,c}, Y_t^{f,k}, \{Y_t^{f,c}(j)\}_{j=0}^1, \{Y_t^{f,k}(j)\}_{j=0}^1, \{Y_t^{m,c}(j)\}_{j=0}^1, \{Y_t^{m,k}(j)\}_{j=0}^1, \{C_t(i)\}_{i=0}^1,$$
$$\{I_t(k)\}_{k=0}^1, \{L_t^{u,c}(i)\}_{i=0}^1, \{L_t^{u,k}(i)\}_{i=0}^1, \{L_t^{y,c}(i)\}_{i=0}^1, \{L_t^{y,k}(i)\}_{i=0}^1, \{K_{t+1}(k)\}_{k=0}^1,$$
$$\{K_t^c(k)\}_{k=0}^1, \{K_t^k(k)\}_{k=0}^1, \{K_t^c(j)\}_{j=0}^1, \{K_t^k(j)\}_{j=0}^1, \{\{L_t^{y,c}(i,j)\}_{i=0}^1\}_{j=0}^1, \{\{L_t^{y,k}(i,j)\}_{i=0}^1\}_{j=0}^1, \Big\}_{t=0}^{\infty}$$

and a sequence of values

$$\Big\{\Pi_t^{p,gdp}, \Pi_t^{p,c}, \Pi_t^{p,k}, \Pi_t^{p,c}(j), \Pi_t^{p,k}(j), \Pi_t^{w,c}, \Pi_t^{w,k}, \Pi_t^{w,c}(i), \Pi_t^{w,k}(i), P_t^k/P_t^c, \{P_t^c(j)/P_t^c\}_{j=0}^1,$$
$$\{P_t^k(j)/P_t^c\}_{j=0}^1, R_t^k/P_t^c, R_t^{k,c}/P_t^c, R_t^{k,k}/P_t^c, W_t^c/P_t^c, W_t^k/P_t^c, \{W_t^c(i)/P_t^c\}_{i=0}^1,$$
$$\{W_t^k(i)/P_t^c\}_{i=0}^1, \{MC_t^c(j)/P_t^c\}_{j=0}^1, \{MC_t^k(j)/P_t^c\}_{j=0}^1, \{Q_t^k(k)/P_t^c\}_{k=0}^1, R_t\Big\}_{t=0}^{\infty}$$

that satisfy the following conditions:

- the final-good producing firms solve (11) for $s = c$ and k;

- all intermediate-good producers $j \in [0,1]$ solve (12), (13), and (15) for $s = c$ and k;
- all capital owners $k \in [0,1]$ solves (17);
- all households $i \in [0,1]$ solve (20);
- all factor markets clear as in (21);
- all intermediate goods markets clear (by construction);
- the two final goods markets clear as in (22);
- the identities given in (23) hold, but are modified slightly to
  $$\frac{W_t^s(i)}{P_t^c} = \frac{\Pi_t^{w,s}(i)}{\Pi_t^{p,c}} \cdot \frac{W_{t-1}^s(i)}{P_{t-1}^c} \text{ and } \frac{W_t^s}{P_t^c} = \frac{\Pi_t^{w,s}}{\Pi_t^{p,c}} \cdot \frac{W_{t-1}^s}{P_{t-1}^c} \text{ for all } i \in [0,1] \text{ and for } s = c, k;$$
- the identities given in (24) hold, although are modified slightly to
  $$\frac{P_t^s(j)}{P_t^c} = \frac{\Pi_t^{p,s}(j)}{\Pi_t^{p,c}} \cdot \frac{P_{t-1}^s(i)}{P_{t-1}^c} \text{ and } \frac{P_t^k}{P_t^c} = \frac{\Pi_t^{p,k}}{\Pi_t^{p,c}} \cdot \frac{P_{t-1}^k}{P_{t-1}^c} \text{ for all } i \in [0,1] \text{ and for } s = c, k;$$
- the monetary authority follows (27) and (28), where the $H_t^{y,gdp}$ and $\Pi_t^{p,gdp}$ are defined by equations (25) and (26).

In solving these problems agents take as given the initial values of K_0 and R_{-1}, and the sequence of exogenous variables

$$\left\{A_t^c, A_t^k, A_t^m, \Gamma_t^{z,c}, \Gamma_t^{z,k}, \Gamma_t^{z,m}, \Theta_t^{y,c}, \Theta_t^{y,k}, \Theta_t^{l,c}, \Theta_t^{l,k}, \Xi_t^b, \Xi_t^l, H_t^{y,gf}\right\}_{t=0}^{\infty}$$

implied by the sequence of shocks

$$\left\{\epsilon_t^{a,c}, \epsilon_t^{a,k}, \epsilon_t^{a,m}, \epsilon_t^{z,c}, \epsilon_t^{z,k}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,c}, \epsilon_t^{\theta,y,k}, \epsilon_t^{\theta,l,c}, \epsilon_t^{\theta,l,k}, \epsilon_t^{\xi,b}, \epsilon_t^{\xi,l}, \epsilon_t^{r}, \epsilon_t^{h,gf}, \epsilon_t^{\pi,gf}\right\}_{t=0}^{\infty}.$$

4 Preparing the Model for Estimation

We make a number of modifications to the variables in the model before estimating it is the following section. First, we simplify the model by noting that all of the individuals within any class of agents—that is, all households, all capital owners, and all intermediate-goods producing firms within the same sector—behave identically to each other. This allows us to drop the i, j, and k indices from all of the model's variables. For the variables pertaining to the decisions of the intermeidate goods producing firms decisions this implies that:

- $Y_t^{f,c}(j) = Y_t^{f,c}$, $Y_t^{f,k}(j) = Y_t^{f,k}$, $Y_t^{m,c}(j) = Y_t^{m,c}$, $Y_t^{m,k}(j) = Y_t^{m,k}$, $K_t^c(j) = K_t^c$, $K_t^k(j) = K_t^k$, $L_t^{s,c}(i,j) = L_t^{s,c}(i)$, $L_t^{s,k}(i,j) = L_t^{s,k}(i)$, $MC_t^c(j)/P_t^c = MC_t^c/P_t^c$, $MC_t^k(j)/P_t^c = MC_t^k/P_t^c$, $P_t^c(j)/P_t^c = 1$, $P_t^k(j)/P_t^c = P_t^k/P_t^c$, $\Pi_t^{p,c}(j) = \Pi_t^{p,c}$, and $\Pi_t^{p,c}(j) = \Pi_t^{p,c}$ for all $j \in [0,1]$.

For the variables pertaining to the decisions of the capital owners this implies that:

- $I_t(k) = I_t$, $K_t^c(k) = K_t^c$, $K_t^k(k) = K_t^k$, and $K_{t+1}(k) = K_{t+1}(k)$ for all $k \in [0,1]$.

For the variables pertaining to the decisions of the households this implies that:

- $C_t(i) = C_t$, $L_t^{u,c}(i) = L_t^{u,c}$, $L_t^{u,k}(i) = L_t^{u,k}$, $L_t^{y,c}(i) = L_t^{y,c}$, $L_t^{y,k}(i) = L_t^{y,k}$, $W_t^c(i)/P_t^c = W_t^c/P_t^c$, $W_t^k(i)/P_t^c = W_t^k/P_t^c$, $\Pi_t^{w,c}(i) = \Pi_t^{w,c}$, and $\Pi_t^{w,k}(i) = \Pi_t^{w,k}$ for all $i \in [0,1]$.

We write the equations of the model so that they are all expressed in terms of stationary variables. The stochastic unit-root Z_t^s technology terms described in equation (3) for $s = c, k, m$ introduce non-stationarities into the model that are divergent across variables. The model variables that must be modified to render them stationary, along with a description of how they are transformed to be made stationary, is given below.

$$\widetilde{Y}_t^{f,c} = \frac{Y_t^{f,c}}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}},\ \widetilde{Y}_t^{f,k} = \frac{Y_t^{f,k}}{Z_t^m Z_t^k},\ \widetilde{Y}_t^{m,c} = \frac{Y_t^{m,c}}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}},\ \widetilde{Y}_t^{m,k} = \frac{Y_t^{m,k}}{Z_t^m Z_t^k},\ \widetilde{I}_t = \frac{I_t}{Z_t^m Z_t^k},$$
$$\widetilde{C}_t = \frac{C_t}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}},\ \widetilde{K}_{t+1} = \frac{K_{t+1}}{Z_t^m Z_t^k},\ \widetilde{K}_t^c = \frac{K_t^c}{Z_{t-1}^m Z_{t-1}^k},\ \widetilde{K}_t^k = \frac{K_t^k}{Z_{t-1}^m Z_{t-1}^k},\ \widetilde{P}_t^k = \frac{P_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha},$$
$$\widetilde{R}_t^k = \frac{R_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}, \widetilde{R}_t^{k,c} = \frac{R_t^{k,c}}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}, \widetilde{R}_t^{k,k} = \frac{R_t^{k,k}}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}, \widetilde{W}_t^c = \frac{W_t^c}{P_t^c}\cdot\frac{1}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}},$$
$$\widetilde{W}_t^k = \frac{W_t^k}{P_t^c}\cdot\frac{1}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}}, \widetilde{Q}_t = \frac{Q_t}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}, \widetilde{MC}_t^c = \frac{MC_t^c}{P_t^c},\ \text{and}\ \widetilde{MC}_t^k = \frac{MC_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}.$$

Equilibrium in the symmetric and stationary model must still satisfy the conditions listed in section 3.9, although some of the conditions—specifically, those implied by **the final goods producing firm's cost minimization problem**, given by (11), and **the first-stage of the intermediate goods producing firm's cost-minimization problem**, given by (12)—are rendered inconsequential by the symmetry of the model.

The second-stage of the intermediate goods producing firm's cost-minimization problem, given by (13), implies the following labor demand schedule, capital demand schedule, and marginal cost expression for sector s:

$$L_t^{y,s} = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{\widetilde{Y}_t^{m,s}}{(A_t^m A_t^s)^{1-\alpha}} \left(\frac{\widetilde{W}_t^s}{\widetilde{R}_t^{k,s}}\right)^{-\alpha}, \qquad \text{for } s = c, k \tag{29}$$

$$\frac{\widetilde{K}_t^s}{\Gamma_t^{y,k}} = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{\widetilde{Y}_t^{m,s}}{(A_t^m A_t^s)^{1-\alpha}} \left(\frac{\widetilde{W}_t^s}{\widetilde{R}_t^{k,s}}\right)^{1-\alpha}, \qquad \text{for } s = c, k. \tag{30}$$

$$\widetilde{MC}_t^s = \frac{1}{(A_t^m A_t^s)^{1-\alpha}} \left(\frac{\widetilde{W}_t^s}{1-\alpha}\right)^{1-\alpha} \left(\frac{\widetilde{R}_t^{k,s}}{\alpha}\right)^{\alpha}, \qquad \text{for } s = c, k. \tag{31}$$

The intermediate goods producing firms' profit-maximization problem, given by (15), yields the sector s supply (or Phillips) curve and an expression that captures the real costs of changing prices:

$$\begin{aligned}\Theta_t^{y,s}\widetilde{MC}_t^s\widetilde{Y}_t^{m,s} &= (1+\sigma^{p,s})\left(\Theta_t^{y,s}-1\right)\widetilde{P}_t^s\widetilde{Y}_t^{m,s} \\ &+ 100\cdot\chi^{p,s}\left(\Pi_t^s - \eta^{p,s}\Pi_{t-1}^s - (1-\eta^{p,s})\Pi_*^s\right)\Pi_t^s\widetilde{P}_t^s\widetilde{Y}_t^{m,s} \\ &- \beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda^c}_{t+1}}{\widetilde{\Lambda}_t^c}\cdot 100\cdot\chi^{p,s}\left(\Pi_{t+1}^s - \eta^{p,s}\Pi_t^s - (1-\eta^{p,s})\Pi_*^s\right)\Pi_{t+1}^s\widetilde{P}_{t+1}^s\widetilde{Y}_{t+1}^{m,s}\right\} \\ &\qquad \text{for } s = c, k,\end{aligned} \tag{32}$$

$$\widetilde{Y}_t^{f,s} = \widetilde{Y}_t^{m,s} - \frac{100\cdot\chi^{p,s}}{2}\left(\Pi_t^{p,s} - \eta^{p,s}\Pi_{t-1}^{p,s} - (1-\eta^{p,s})\Pi_*^{p,s}\right)^2\widetilde{Y}_t^{m,s} \quad \text{for } s = c, k. \tag{33}$$

The capital owner's profit-maximization problem, given by (17), yields the following conditions for the supply of capital

$$\widetilde{Q}_t = \beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda}_{t+1}^c}{\widetilde{\Lambda}_t^c}\cdot\frac{1}{\Gamma_{t+1}^{y,k}}\left(\widetilde{R}_{t+1}^k + (1-\delta)\widetilde{Q}_{t+1}\right)\right\} \tag{34}$$

$$\widetilde{R}_t^{k,c} = \widetilde{R}_t^k\left[1 + 100\cdot\chi^k\left(\frac{\widetilde{K}_t^c}{\widetilde{K}_t^k} - \eta^k\frac{\widetilde{K}_{t-1}^c}{\widetilde{K}_{t-1}^k} - (1-\eta^k)\frac{\widetilde{K}_*^c}{\widetilde{K}_*^k}\right)\frac{\widetilde{K}_t}{\widetilde{K}_t^c}\right] \tag{35}$$

$$\widetilde{R}_t^{k,k} = \widetilde{R}_t^k\left[1 - 100\cdot\chi^k\left(\frac{\widetilde{K}_t^c}{\widetilde{K}_t^k} - \eta^k\frac{\widetilde{K}_{t-1}^c}{\widetilde{K}_{t-1}^k} - (1-\eta^k)\frac{\widetilde{K}_*^c}{\widetilde{K}_*^k}\right)\frac{\widetilde{K}_t}{\widetilde{K}_t^k}\right] \tag{36}$$

$$\begin{aligned}\widetilde{P}_t^k = \widetilde{Q}_t&\left[1 - 100\cdot\chi^i\left(\frac{\widetilde{I}_t - \eta^i\widetilde{I}_{t-1} - (1-\eta^i)\widetilde{I}_*}{\widetilde{K}_t}\cdot\Gamma_t^{y,k}\right)\right] \\ &+\beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda}_{t+1}^c}{\widetilde{\Lambda}_t^c}\cdot\widetilde{Q}_{t+1}\cdot 100\cdot\chi^i\cdot\eta^i\cdot\Gamma_{t+1}^{y,k}\left(\frac{\widetilde{I}_{t+1} - \eta^i\widetilde{I}_t - (1-\eta^i)\widetilde{I}_*}{\widetilde{K}_{t+1}}\cdot\Gamma_{t+1}^{y,k}\right)\right\}\end{aligned} \tag{37}$$

as well as an implied expression for investment demand and a market clearing condition for capital:

$$\widetilde{K}_{t+1} = (1-\delta)\frac{\widetilde{K}_t}{\Gamma_t^{y,k}} + \widetilde{I}_t - \frac{100\cdot\chi^i}{2}\left(\frac{\widetilde{I}_t - \eta^i \widetilde{I}_{t-1} - (1-\eta^i)\widetilde{I}_*}{\widetilde{K}_t}\cdot\Gamma_t^{y,k}\right)^2 \frac{\widetilde{K}_t}{\Gamma_t^{y,k}} \tag{38}$$

$$\widetilde{K}_t^c + \widetilde{K}_t^k = \widetilde{K}_t - \frac{100\cdot\chi^k}{2}\left(\frac{\widetilde{K}_t^c}{\widetilde{K}_t^k} - \eta^k\frac{\widetilde{K}_{t-1}^c}{\widetilde{K}_{t-1}^k} - (1-\eta^k)\frac{\widetilde{K}_*^c}{\widetilde{K}_*^k}\right)^2 \frac{\widetilde{K}_t^k}{\widetilde{K}_t^c}\cdot\widetilde{K}_t \tag{39}$$

The household's utility-maximization problem, given by (20), implies the following expression for consumption demand and labor supply as well as an expression that captures the real cost of changing wages and the composition of labor:

$$\widetilde{\Lambda}_t^c = \beta R_t \mathcal{E}_t\left[\widetilde{\Lambda}_{t+1}^c \cdot \frac{1}{\Pi_{t+1}^{p,c}\Gamma_{t+1}^{y,c}}\right] \tag{40}$$

$$\begin{aligned}
\Theta_t^{l,s}\frac{\bar{\Lambda}_t^{l,s}}{\widetilde{\Lambda}_t^c}L_t^{u,s} = {} & (1+\sigma^{w,s})\left(\Theta_t^{l,s}-1\right)\widetilde{W}_t^s L_t^{u,s} \\
& + \frac{\bar{\Lambda}_t^{l,s}}{\widetilde{\Lambda}_t^c} 100\cdot\chi^{w,s}\left(\Pi_t^{w,s} - \eta^{w,s}\Pi_{t-1}^{w,s} - (1-\eta^{w,s})\Pi_*^{w,s}\right)\Pi_t^{w,s}\widetilde{W}_t^s L_t^{u,s} \\
& - \beta\mathcal{E}_t\left\{\frac{\widetilde{\Lambda}_{t+1}^c}{\widetilde{\Lambda}_t^c}\cdot\frac{\bar{\Lambda}_t^{l,s}}{\widetilde{\Lambda}_t^c}100\cdot\chi^{w,s}\left(\Pi_{t+1}^{w,s} - \eta^{w,s}\Pi_t^{w,s} - (1-\eta^{w,s})\Pi_*^{w,s}\right)\Pi_{t+1}^{w,s}\widetilde{W}_t^s L_{t+1}^{u,s}\right\} \\
& \text{for } s = c,k.
\end{aligned} \tag{41}$$

$$\begin{aligned}
L_t^{y,s} = {} & L_t^{u,s} - \frac{100\cdot\chi^{w,s}}{2}\left(\Pi_t^{w,s} - \eta^{w,s}\Pi_{t-1}^{w,s} - (1-\eta^{p,s})\Pi_*^{w}\right)^2 L_t^{u,s} \\
& - \frac{L_*^{u,s}}{L_*^{u,c}+L_*^{u,k}}\cdot\frac{100\cdot\chi^l}{2}\left(\frac{L_t^{u,c}}{L_t^{u,k}} - \eta^l\frac{L_{t-1}^{u,c}}{L_{t-1}^{u,k}} - (1-\eta^l)\frac{L_*^{u,c}}{L_*^{u,k}}\right)^2\frac{L_t^{u,k}}{L_t^{u,c}}, \\
& \text{for } s = c,k.
\end{aligned} \tag{42}$$

where the normalized marginal utility of consumption, $\widetilde{\Lambda}_t^c$, is given by

$$\widetilde{\Lambda}_t^c = \Xi_t^b\left(\widetilde{C}_t - h\widetilde{C}_{t-1}/\Gamma_t^{y,c}\right)^{-1}, \tag{43}$$

and $\bar{\Lambda}_t^{l,c}$ and $\bar{\Lambda}_t^{l,c}$ are related to the marginal ***dis***-utilities of labor, $\Lambda_t^{l,c}$ and $\Lambda_t^{l,k}$, according to:

$$\bar{\Lambda}_t^{l,c} L_t^{u,c} = \underbrace{\varsigma \Xi_t^l \left(L_t^{u,c} + L_t^{u,k}\right)^{\nu}}_{\Lambda_t^{l,c}} L_t^{u,c} + \frac{\widetilde{\Lambda}_t^c}{P_t^c}\left(W_t^c L_t^c + W_t^k L_t^k\right) 100 \cdot \chi^l \left(\frac{L_t^{u,c}}{L_t^{u,k}} - \eta^l \frac{L_{t-1}^{u,c}}{L_{t-1}^{u,k}} - (1-\eta^l)\frac{L_*^{u,c}}{L_*^{u,k}}\right) \tag{44}$$

$$\Lambda^{\bar{l},k}{}_t L_t^{u,k} = \underbrace{\varsigma \Xi_t^l \left(L_t^{u,c} + L_t^{u,k}\right)^{\nu}}_{\Lambda_t^{l,c}} L_t^{u,k} - \frac{\widetilde{\Lambda}_t^c}{P_t^c}\left(W_t^c L_t^c + W_t^k L_t^k\right) 100 \cdot \chi^l \left(\frac{L_t^{u,c}}{L_t^{u,k}} - \eta^l \frac{L_{t-1}^{u,c}}{L_{t-1}^{u,k}} - (1-\eta^l)\frac{L_*^{u,c}}{L_*^{u,k}}\right) \tag{45}$$

The stationary final goods market clearing conditions are given by:

$$\widetilde{Y}_t^c = \widetilde{C}_t \quad \text{and} \quad \widetilde{Y}_t^k = \widetilde{I}_t. \tag{46}$$

And **the stationary wage and price level and inflation identities** are given by:

$$\widetilde{W}_t^c = \frac{\Pi_t^{w,c}}{\Pi_t^{p,c}} \cdot \frac{1}{\Gamma_t^{y,c}} \cdot \widetilde{W}_{t-1}^c, \quad \widetilde{W}_t^k = \frac{\Pi_t^{w,k}}{\Pi_t^{p,c}} \cdot \frac{1}{\Gamma_t^{y,c}} \cdot \widetilde{W}_{t-1}^k, \quad \text{and} \quad \widetilde{P}_t^k = \frac{\Pi_t^{w,s}}{\Pi_t^{p,c}} \cdot \frac{\Gamma_t^{y,s}}{\Gamma_t^{y,c}} \cdot \widetilde{P}_{t-1}^k. \tag{47}$$

Equations (27) and (28), that describe the monetary authorities' policy feedback rule, are unchanged in the stationary model, and $\Pi_t^{p,gdp}$ is still given by equation (26). The expression for $H_t^{y,gdp}$, however, is re-written as:

$$\begin{aligned} H_t^{y,gdp} = {} & \left(\Gamma_t^{y,c} \cdot \widetilde{Y}_t^c / \widetilde{Y}_{t-1}^c\right)^{\frac{1}{2} \cdot \frac{P_t^c Y_t^c}{P_t^c Y_t^c + P_t^k Y_t^k + P_t^{gf} Y_t^{gf}} + \frac{1}{2} \cdot \frac{P_{t-1}^c Y_{t-1}^c}{P_{t-1}^c Y_{t-1}^c + P_{t-1}^k Y_{t-1}^k + P_{t-1}^{gf} Y_{t-1}^{gf}}} \\ & \times \left(\Gamma_t^{y,k} \cdot \widetilde{Y}_t^k / \widetilde{Y}_{t-1}^k\right)^{\frac{1}{2} \cdot \frac{P_t^k Y_t^k}{P_t^c Y_t^c + P_t^k Y_t^k + P_t^{gf} Y_t^{gf}} + \frac{1}{2} \cdot \frac{P_{t-1}^k Y_{t-1}^k}{P_{t-1}^c Y_{t-1}^c + P_{t-1}^k Y_{t-1}^k + P_{t-1}^{gf} Y_{t-1}^{gf}}} \\ & \times \left(H_t^{y,gf}\right)^{\frac{1}{2} \cdot \frac{P_t^{gf} Y_t^{gf}}{P_t^c Y_t^c + P_t^k Y_t^k + P_t^{gf} Y_t^{gf}} + \frac{1}{2} \cdot \frac{P_{t-1}^c Y_{t-1}^c}{P_{t-1}^c Y_{t-1}^c + P_{t-1}^k Y_{t-1}^k + P_{t-1}^{gf} Y_{t-1}^{gf}}}. \end{aligned} \tag{48}$$

4.1 Equilibrium

Equilibrium in the symmetric and stationary model can thus be defined as an allocation:

$$\left\{H_t^{y,gdp}, \widetilde{Y}_t^{f,c}, \widetilde{Y}_t^{f,k}, \widetilde{Y}_t^{m,c}, \widetilde{Y}_t^{m,k}, \widetilde{C}_t, \widetilde{I}_t, L_t^{u,c}, L_t^{u,k}, L_t^{y,c}, L_t^{y,k}, \widetilde{K}_{t+1}, \widetilde{K}_t^c, \widetilde{K}_t^k\right\}_{t=0}^{\infty}$$

and a sequence of values

$$\left\{\Pi_t^{p,gdp}, \Pi_t^{p,c}, \Pi_t^{p,k}, \Pi_t^{w,c}, \Pi_t^{w,k}, \widetilde{P}_t^k, \widetilde{R}_t^k, \widetilde{R}_t^{k,c}, \widetilde{R}_t^{k,k}, \widetilde{W}_t^c, \widetilde{W}_t^k, \widetilde{MC}_t^c, \widetilde{MC}_t^k, \widetilde{Q}_t^k, R_t\right\}_{t=0}^{\infty}.$$

that satisfy equations (26) to (48), taking as given the initial values of K_0 and R_{-1}, and the sequence of exogenous variables

$$\left\{A_t^c, A_t^k, A_t^m, \Gamma_t^{z,c}, \Gamma_t^{z,k}, \Gamma_t^{z,m}, \Theta_t^{y,c}, \Theta_t^{y,k}, \Theta_t^{l,c}, \Theta_t^{l,k}, \Xi_t^b, \Xi_t^l, H_t^{y,gf}\right\}_{t=0}^{\infty}$$

implied by the sequence of shocks

$$\left\{\epsilon_t^{a,c}, \epsilon_t^{a,k}, \epsilon_t^{a,m}, \epsilon_t^{z,c}, \epsilon_t^{z,k}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,c}, \epsilon_t^{\theta,y,k}, \epsilon_t^{\theta,l,b}, \epsilon_t^{\theta,l,k}, \epsilon_t^{\xi,c}, \epsilon_t^{\xi,l}, \epsilon_t^{r}, \epsilon_t^{h,gf}\right\}_{t=0}^{\infty}.$$

5 Estimation

This sections describes in detail the empirical approach chosen in this paper. We first make a number of assumptions to the model; these are outlined in section 6.1. We solve the log-linear approximation to the modified DSGE model; the solution is given in section 5.2.[2] This resulting dynamical system is cast under its state space representation for a determined set of (in our case nine) observable variables. These variables, their data sources, and their relation to the variables in the model are described in section 5.3. We then use the kalman filter to evaluate the likelihood of the observed variables. We form the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs and information we have about them; our priors are listed in section 5.4. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods to draw from it[3].

5.1 Assumptions about the Structure of the Estimated Model

Prior to estimation the following assumption were made about the specifications of the model.

- As in Smets and Wouters [2004], we reduce the markup shocks, $\theta_t^{y,c}$, $\theta_t^{y,k}$, $\theta_t^{l,c}$, and $\theta_t^{l,k}$ to white noise processes. I addition we assume that there is only one mark-up shock process for the overall labor market, so that $\theta_t^l = \theta_t^{l,c} = \theta_t^{l,k}$.

[2]We do this using the package `gensys.m` written by Chris Sims to obtain this solution.

[3]We refer the reader to the appendix for a more detailed presentation of the MCMC methods.

- The parameters measuring the degree of inter-sectorial adjusment costs for capital, χ^k, has been set to zero.[4]

- The estimated version of the model possesses only two technology shocks, a permanent shock to the level of total factor productivity, represented by $\Gamma_t^{z,m}$, and a permanent shock to the level of investment-specific technology, represented by $\Gamma_t^{z,k}$.[5] This eliminates from the model the following shocks $\{\epsilon_t^{a,c}, \epsilon_t^{a,k}, \epsilon_t^{a,m}, \epsilon_t^{z,c}\}$ which implies that $\{A_t^c, A_t^k, A_t^m, \Gamma_t^{z,c}\} = \{1, 1, 1, \Gamma_*^{z,c}\}$.

- Finally, we have imposed measurement error processes, denoted η_t, for all of the observables except for the nominal interest rate and the aggregate hours series. In all cases, the measurement errors explain less that 5 percent of the observed series.[6]

At this stage of the project, we do not utilize the full possibilities of the multi-sector approach. We assume (in some sense unrealisticaly) that many features of the sectors are identical (depreciation rate of capital, indexation coefficients, etc.).

5.2 Solution to the Log-linearized Model

The solution to the log-linearized version of our model is given by:

$$\underbrace{\left[\alpha_{\mathbf{t}}\right]}_{43\times1} = \underbrace{\left[\mathbf{T}\right]}_{43\times43} \cdot \underbrace{\left[\alpha_{\mathbf{t-1}}\right]}_{43\times1} + \underbrace{\left[\mathbf{R}\right]}_{43\times9} \cdot \underbrace{\left[\epsilon_t\right]}_{9\times1} \tag{49}$$

where

$$\alpha_{\mathbf{t}} = \Big[h_t^{y,gdp}, y_t^{f,c}, y_t^{f,k}, y_t^{m,c}, y_t^{m,k}, c_t, i_t, l_t^{u,c}, l_t^{u,k}, l_t^{y,c}, l_t^{y,k}, k_{t+1}, k_t^c, k_t^k, \pi_t^{p,gdp}, \pi_t^{p,c}, \pi_t^{p,k}, \\ \pi_t^{w,c}, \pi_t^{w,k}, p_t^k, r_t^k, r_t^{k,c}, r_t^{k,k}, w_t^c, w_t^k, mc_t^c, mc_t^k, q_t^k, r_t, \gamma_t^{z,k}, \gamma_t^{z,m}, \theta_t^{y,c}, \theta_t^{y,k}, \theta_t^l, \xi_t^b, \xi_t^l, h_t^{y,gf}\Big]' \tag{50}$$

and

$$\epsilon_t = \left[\epsilon_t^{z,k}, \epsilon_t^{z,m}, \epsilon_t^{\theta,y,c}, \epsilon_t^{\theta,y,k}, \epsilon_t^{\theta,l}, \epsilon_t^{\xi,b}, \epsilon_t^{\xi,l}, \epsilon_t^{r}, \epsilon_t^{h,gf}\right]'. \tag{51}$$

[4] Attempts to estimate the inter-sectorial adjustment cost coefficient associated with capital, χ^k, have been unfruitfull in the sense that, based on the current choice of model and data, the best specification is the one assuming that χ^k is equal to 0.

[5] The complexity of the production structure presented in section 2 relative to the number of observables makes several of the technology shocks redundant and render the model prone to identification issues.

[6] There is one exception which is consumption growth; issues associated with the ability of DSGE models to explain consumption are also observed in Smets and Wouters [2004].

5.3 Data

The model is estimated off nine data series. The series and their sources are:

1. Nominal gross domestic product (GDP_t^n), from the BEA's National Income and Product Accounts.

2. Nominal consumption expenditure on (CNS_t^n), which is equal to the linear aggregation of nominal personal consumption expenditures on nondurables goods and services, both from the BEA's National Income and Product Accounts.

3. Nominal investment expenditure on (CDI_t^n), which is equal to the linear aggregation of nominal personal consumption expenditures on durable goods and nominal gross private domestic investment, both from the BEA's National Income and Product Accounts.

4. GDP price inflation (GDP_t^π) from the National Income and Product Accounts.

5. Consumption price inflation (CNS_t^π), which is equal to the chain weighted aggregation of the rates of price inflation of consumer nondurables goods and consumer services, both from the BEA's National Income and Product Accounts.

6. Investment price inflation (CDI_t^π), which is equal to the chain weighted aggregation of the rates of price inflation on consumption durable goods and private domestic investment goods, both from the BEA's National Income and Product Accounts.

7. Hours (HRS_t), which is equal to hours of all persons in the non-farm business sector from the BLS' Productivity and Cost release.

8. Wage inflation (WG_t^π), which is equal to real compensation per hours in the non-farm business sector from the BLS' Productivity and Cost release.

9. The federal funds rate (RFF_t), that is the policy rate.

The relationships between the series used to estimate the model and the variables from the log-linearized version of the model are:

$$\ln\left(GDP_t^n/GDP_{t-1}^n\right) = \ln \Gamma_*^{y,gdp}\Pi_*^{p,gdp} + h_t^{gdp} + \pi_t^{gdp} \tag{52}$$

$$\ln\left(CNS_t^n/CNS_{t-1}^n\right) = \ln \Gamma_*^{y,c}\Pi_*^{p,c} + c_t - c_{t-1} + \pi_t^{p,c} + \gamma_t^{z,m} + \alpha\gamma_t^{z,k} + (1-\alpha)\gamma_t^{z,c} \tag{53}$$

$$\ln\left(CDI_t^n/CDI_{t-1}^n\right) = \ln \Gamma_*^{y,k}\Pi_*^{p,k} + i_t - i_{t-1} + \pi_t^{p,k} + \gamma_t^{z,m} + \gamma_t^{z,k} \tag{54}$$

$$\ln GDP_t^{\pi} = \ln \Pi_*^{p,gdp} + \pi_t^{p,gdp} \tag{55}$$

$$\ln CNS_t^{\pi} = \ln \Pi_*^{p,c} + \pi_t^{p,c} \tag{56}$$

$$\ln CDI_t^{\pi} = \ln \Pi_*^{p,k} + \pi_t^{p,k} \tag{57}$$

$$\ln WG_t^{\pi} = \ln \Pi_*^{w} + \frac{L_*^{y,c}}{L_*^{y,c}+L_*^{y,k}}\cdot \pi_t^{w,c} + \frac{L_*^{y,k}}{L_*^{y,c}+L_*^{y,k}}\cdot \pi_t^{w,k} \tag{58}$$

$$\ln R_t = \ln R_* + r_t \tag{59}$$

$$\ln HRS_t = \ln(L_*^c + L_*^k) + \frac{L_*^{y,c}}{L_*^{y,c}+L_*^{y,k}}\cdot l_t^{y,c} + \frac{L_*^{y,k}}{L_*^{y,c}+L_*^{y,k}}\cdot l_t^{y,k} \tag{60}$$

All of the steady-state values given in equations (52) to (59) are functions of the parameters of the model (or are themselves parameters of the model, as in the case of $\Pi_*^{p,c}$). Specifically,

$$\begin{aligned}
\Gamma_*^{y,c} &= \Gamma_*^{z,m}(\Gamma_*^{z,k})^{\alpha}(\Gamma_*^{z,c})^{1-\alpha},\\
\Gamma_*^{y,k} &= \Gamma_*^{z,m}\Gamma_*^{z,k},\\
\Gamma_*^{y,gdp} &= (\Gamma_*^{y,c})^{\frac{P_*^cY_*^c}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}}\left(\Gamma_*^{y,k}\right)^{\frac{P_*^kY_*^k}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}}\left(H_*^{y,gf}\right)^{\frac{P_*^{gf}Y_*^{gf}}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}},\\
\Pi_*^{p,k} &= \Pi_*^{p,c}(\Gamma_*^{y,c}/\Gamma_*^{y,k}) = \Pi_*^{p,c}(\Gamma_*^{z,c}/\Gamma_*^{z,k})^{1-\alpha},\\
\Pi_*^{p,gdp} &= (\Pi_*^{p,c})^{\frac{P_*^cY_*^c}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}}\left(\Pi_*^{p,k}\right)^{\frac{P_*^kY_*^k}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}}\left(\Pi_*^{p,hf}\right)^{\frac{P_*^{gf}Y_*^{gf}}{P_*^cY_*^c+P_*^kY_*^k+P_*^{gf}Y_*^{gf}}},\\
\Pi_*^{w,s} &= \Pi_*^{w} = \Pi_*^{p,c}\Gamma_*^{y,c} = \Pi_*^{p,c}\Gamma_*^{z,m}(\Gamma_*^{z,k})^{\alpha}(\Gamma_*^{z,c})^{1-\alpha},\\
R_* &= (1/\beta)\Pi_*^{p,c}\Gamma_*^{z,m}(\Gamma_*^{z,k})^{\alpha}(\Gamma_*^{z,c})^{1-\alpha},
\end{aligned}$$

and

$$L_*^c + L_*^k = \left[\left(\frac{1+\mathcal{B}}{\mathcal{A}^{\frac{\alpha}{1-\alpha}}}\right)\frac{1-\alpha}{\varsigma}\left(\frac{\alpha}{\Gamma_*^{y,k}/\beta-(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{h}{\Gamma_*^{y,c}}\right)^{-1}\right]^{\frac{1}{\upsilon+1}},$$

where

$$\mathcal{A} = \frac{\alpha}{\Gamma_*^{y,k}/\beta-(1-\delta)} \quad\text{and}\quad \mathcal{B} = \left[\frac{\Gamma_*^{y,k}/\beta-(1-\delta)}{\alpha}\cdot\frac{\Gamma_*^{y,k}}{\Gamma_*^{y,k}-(1-\delta)}-1\right]^{-1}.$$

Equations (52) to (59), with the steady-state values listed above imposed, are the measurement equations of our model, which can be summarized as:

$$\underbrace{\left[\ DATA_t\ \right]}_{9\times1}=\underbrace{\left[\ \mathbf{Z}\ \right]}_{9\times43}\cdot\underbrace{\left[\ \alpha_{\mathbf{t}}\ \right]}_{43\times1}+\underbrace{\left[\ \mathbf{d}\ \right]}_{9\times1}+\underbrace{\left[\ \mathbf{H}\ \right]}_{9\times9}\cdot\underbrace{\left[\ \eta_t\ \right]}_{9\times1} \tag{61}$$

where

$$\begin{aligned} DATA_t = \big[&\ln\left(GDP_t^n/GDP_{t-1}^n\right), \ln\left(CNS_t^n/CNS_{t-1}^n\right), \ln\left(CDI_t^n/CDI_{t-1}^n\right), \\ &\ln GDP_t^\pi, \ln CNS_t^\pi, \ln CDI_t^\pi, \ln WG_t^\pi, \ln R_t, \ln HRS_t\big]', \end{aligned} \tag{62}$$

$$\begin{aligned} \mathbf{d} = \Big[&\ln\Gamma_*^{y,gdp}\Pi_*^{p,gdp}, \ln\Gamma_*^{y,c}\Pi_*^{p,c}, \ln\Gamma_*^{y,k}\Pi_*^{p,k}, \ln\Pi_*^{y,gdp}, \ln\Pi_*^{y,c}, \ln\Pi_*^{y,k}, \\ &\ln\Pi_*^{w}, \ln R_*, \ln(L_*^c+L_*^k)\Big]', \end{aligned} \tag{63}$$

$$\eta_t = \left[\ \eta^{gdp,n}\ ,\ \eta^{cns,n}\ ,\ \eta^{cdi,n}\ ,\ \eta^{gdp,\pi}\ ,\ \eta^{cns,\pi}\ ,\ \eta^{cdi,\pi}\ ,\ \eta^{wg,\pi}\ ,\ 0\ ,\ 0\ \right], \tag{64}$$

and $\alpha_{\mathbf{t}}'$ is described as in equation (50). The matrix $\mathbf{H}$ is a diagonal matrix for which

$$Diag(\mathbf{H})=\left[\ \sigma^{gdp,n}\ ,\ \sigma^{cns,n}\ ,\ \sigma^{cdi,n}\ ,\ \sigma^{gdp,\pi}\ ,\ \sigma^{cns,\pi}\ ,\ \sigma^{cdi,\pi}\ ,\ \sigma^{wg,\pi}\ ,\ 0\ ,\ 0\ \right]. \tag{65}$$

5.4 Prior Distribution

Some of the model's coefficients were determined and their values fixed prior to the estimation. We based our choices on considerations about the informativeness of the data, identification issues and overparameterization. The discount factor β is equal to 0.995. The depreciation rate of capital has been set to 0.025 in both sectors. We assume the same steady-state markup level across the monopolistic sectors of the economy; $\Theta_*^{p,y,c}$, $\Theta_*^{p,y,k}$ and Θ_*^{w} are equal to 7. We assume that only growth in investment is costly for both sectors, i.e. $\eta^i = 1$. The capital share of income at the steady state corresponds to a α equal to 0.3 for both sectors. The average nominal share of autonomous spending over total GDP has been set to 0.2.

Table 1 presents our assumptions about the prior distributions of the estimated parameters. The joint prior distribution is the product of independent densitities. The prior densities of the adjustment costs parameters, χ^p, χ^i, χ^H and χ^w are identical and are described by a gamma distribution with mean 2 and standard deviation 1. The corresponding indexation coefficients, η^p, η^i, η^H and η^w, follow beta distributions that share the same mean, 0.5, and standard deviation, 0.22.

The confidence intervals implied by the prior distributions of the policy rule parameters are fairly wide, leaving an important role to the data in the determination of these coefficients.

6 Results

6.1 Posterior Distribution

Once properly weighted by their respective factors, our model indicates that the most important nominal rigidities are those associated with the wage inflation adjustment costs. These results are consistent with those presented in Levin *et al.* [2005] for an one-sector model of the US economy. Both adjusmtent mechanisms associated with price and wage inflation indicate that deviations from the steady-state levels are more important that those derived from lagged values, leaving a lesser role to indexation.

Our estimates of the parameters characterizing the household's utility function are close to the bounds of the parameter space. The mass of the posterior distribution for the elasticity of labor substitution, ν, leans towards its lower range. The posterior density of the degree of habit formation, h, implies estimtates that are tightly concentrated to values just below the "*a priori*" upper bound 1. We consider this an undesireable feature of the model and in future drafts will consider modifications to the model—such as non-logarithmic preferences, the presence of rule of thrumb consumers, and differnt assumptions regarding measurement errors—that could lower the estimated degree of habit persistence.

The posterior distribution also shows that the permanent technology shocks have explained, on average, a similar share of the growth observed in real consumption and real total output. We notice that the persistence of the investment-specific technology shock, $\rho^{z,k}$, is much higher—0.803 compared with 0.544 when evaluated at the mode—than that of the total-factor productivity shock, $\rho^{z,c}$.

6.2 Variance Decomposition

Table 2 displays the shocks' contribution to the movements observed at different horizons in the real observables used in the estimation. Table 3 presents a similar decomposition for the three prices of section 5.3 and the nominal interest rate.

6.3 Impulse Responses

Figures 2 through 5 report the impulse responses for the model's key variables (that is, the variables for which we use data) following innovations to the model's nine primitive shocks. The responses generated by the model to a monetary policy shock, a sector neutral technology shock, and a capital specific technology shock all conform with conventional thinking. Following a monetary policy shock, real consumption, real investment, GDP growth, aggregate hours, and all measures of inflation decline and since real investment (which also includes consumer durables) is more interest sentsitive than (nondurables and services) consumption it shows a notably larger declines. Following both of the technology shocks investment increases strongly (and overshoots its long-run level) to bring the capital stock inline with the new higher level of technology. Consumption in contrast increases slowly to its long run level. Price inflation in both the consumption and capital goods sectors slows, while wage inflation picks up, thereby bring the real wage in to line with its new higher long-run level.

6.4 Implied Paths

Figure 6 compares the one-step ahead DSGE forecast to the actual observations from the data. Figure 7 shows the 95 percent confidence bands and the median of the smoothed paths of the persistent structural shocks. Our model implies that the recession of 1991 was caused by a decline in both technology shocks, though that of the TFP is more acute. [To be completed.]

7 Policy Evaluation

This section presents different exercices related to the use of our estimated DSGE model for monetary policy. First, we investigate the usefullness of key concepts offered by the New Keynesian literature in the context of our model. Second, we assess the welfare implications of simple monetary policy rules under a popular quadractic loss function. We pay a particular attention to the usefulness of different measures of real activity, beside GDP inflation, as relevant indicators for the policymakers.

7.1 Flexible-Price Equilibrium

This section offers a preliminary look at some key concepts put forward by the New Keynesian literature as normative instruments for the conduct of monetary policy.[7]

The first graph of figure 8 displays a key measure of the output gap used by FRB/US in its simulations and forecasts. The second graph shows our model's measure of output gap as defined above[8]. The correlation between the flex-price gap and the FRB/US measure is 0.36. Simple calculations show the former leads the latter since lagging the flex-price measure by one year increases the correlation to 0.6.

Figure 9 shows the estimated and flex-price real rates, both expressed as percentage deviation from steady-state. We notice that the model's natural real is a lot more volatile than the one implied from the estimated model. The lack of persistence and the relatively large movements displayed by the natural rate can be found in the extremly high estimated degree of habit formation. Our model implies that as h goes to one, larger changes in the real rates are needed to induce movements in consumption of equal magnitude.

Figure 10 plots the paths of the natural rate gap, i.e. the estimated real rate minus the natural rate, and the nominal interest rate. The movements we observe in the natural rate gap appears consistent with the conventional views: the 80's tigh policy stance is associated with positive gaps while the expansionist views following the 1991 and 2001 recessions correspond to fairly negative gaps.

7.2 Simple Policy Rules

The welfare criterion is the standard quadractic loss function:

$$E\left(\pi_t^{p,gdp}\right)^2 + \Lambda_Y E\left(\ln(Y_t^{gdp}/Y_{t-1}^{gdp}) - \Gamma_*^{y,gdp}\right)^2 + \Lambda_R E(\Delta r_t)^2. \tag{66}$$

[7] Our definition of the output gap is slightly different than that find in Nelson and Neiss [2003] and Smets and Wouters [2004]. The counterfactual equilibrium still corresponds to an economy where the nominal rigidities are absent. However, in the spirit of Woodford [2004], we assume that the agents make decisions based on the actual realizations of the state variables.

[8] As opposed to the one-sector model, there is no total output concept in our model and the divisa index only characterizes, in a stationary environment, the growth of total output. For this reason, the output gap considered in this section is constructed as the sum, weighted by their nominal shares, of the two sectorial gaps.

This loss function, as opposed to a welfare-based criterion, makes it easier to compare the normative implications of our model to the other models currently in use at the Board such as FRB/US (although we will in subsequent drafts consider both criteria). Real GDP growth and the GDP deflator are the measures of output growth and inflation that enter the loss function 66.

We assess simple policy rules of the form:

$$r_t = \rho_R r_{t-1} + (1-\rho_R)(a_1 \pi_t^{p,gdp} + a_2 x_t) \tag{67}$$

and

$$r_t = r_{t-1} + (a_1 \pi_t^{p,gdp} + a_2 x_t) \tag{68}$$

where x_t is always one of four possible policy indicators: (i) real output, growth, (ii) wage inflation, (iii) the output gap, and (iv) the natural real rate. The last two indicators are unobserved and are defined in section 7.1. The first two are directly observed from the data by the policymakers and the private sector.

We assume the values $\Lambda_Y = 1$ and $\Lambda_R = 1$[9]. Figure 11 presents the welfare surfaces—that is, inverted loss functions—derived from the rule given in equation 67 for the cases when x_t is one of the two observable indicators and ρ_R is equal to its posterior mode value, 0.852. Figure 12 presents the welfare surfaces corresponding to the two unobserved indicators while ρ_R stays the same. A welfare level of 0 indicates that the implied equilibrium is not unique and/or does not exist. The economy achieves its highest level of welfare when the output gap is used in the policy rule. The corresponding welfare surface favors a moderate long-run response to inflation, $a_1 = 1.76$, and strong adjustments to movements in the output gap, $a_2 = 2.00$. The real ouput growth measure produces the lowest welfare levels.

Figures 13 and Figure 14 show similar welfare surfaces for the rule 68, which is often refered by the literature as a "first-difference" rule. For any of the real activity measure, the welfare level associated to the best rule is always higher than its counterpart under the estimated degree of interest rate smoothing, making a strong case for the "first-difference" rule. However, the best indicator under that rule is the output growth, which is a complete reversal from the stationary case. The best first-difference rule is

$$r_t = r_{t-1} + (0.51 \pi_t^{p,gdp} + 0.5 \delta y_t^{gdp}), \tag{69}$$

[9]We also look at other values, such as $\Lambda_Y = 1$ and $\Lambda_R = 0$, but we found no major differences in the normative conclusions derived from the alternative specifications.

in which policymakers respond with equal weights to the two indicators.

8 Conclusion

To be completed.

References

Levin, A., Onatski, A., Williams, J., and Williams, N. (2005). Monetary policy under uncertainty in micro-founded macroecometric models. *Manuscript*.

Nelson, E. and Neiss, K. (2003). The real-interest-rate gap as an inflation indicator. *Macroeconomic Dynamics*, **7(2)**, 353–370.

Smets, F. and Wouters, R. (2004). Shocks and frictions in the us busines cycles: A bayesian dsge approach. Manuscript.

Woodford, M. (2004). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Table 1: Prior and Posterior Distribution

	Prior Distribution			Posterior Distribution				
Parameter	Type	Mean	S.D.	Mode	S.D.	10th perc.	50th perc.	90th perc.
h	B	0.500	0.224	0.989	0.006	0.981	0.989	0.994
ν	G	2.000	1.000	0.266	0.353	0.176	0.511	1.095
$\phi^{\pi,gdp}$	N	2.000	1.000	2.871	0.379	2.441	2.853	3.342
$\phi^{h,gdp}$	N	0.500	0.400	0.165	0.039	0.119	0.166	0.220
$\phi^{\triangle\pi,gdp}$	N	0.500	0.400	-0.201	0.072	-0.287	-0.195	-0.100
$\phi^{\triangle h,gdp}$	N	0.500	0.400	-0.105	0.035	-0.150	-0.104	-0.059
$\chi_{p,c}$	G	2.000	1.000	3.652	1.256	3.183	4.467	6.455
χ_{l}	G	2.000	1.000	1.632	1.251	1.301	2.522	4.461
$\chi_{w,c}$	G	2.000	1.000	4.191	1.825	3.718	5.475	7.923
$\chi_{i,c}$	G	2.000	1.000	0.752	0.382	0.633	0.945	1.470
η^{l}	B	0.500	0.224	0.349	0.186	0.162	0.394	0.660
$\eta^{p,c}$	B	0.500	0.224	0.290	0.112	0.184	0.319	0.474
$\eta^{w,c}$	B	0.500	0.224	0.300	0.098	0.160	0.286	0.410
ρ_{R}	B	0.750	0.112	0.852	0.023	0.821	0.851	0.879
$\rho^{\xi,b}$	B	0.750	0.112	0.732	0.091	0.621	0.747	0.860
$\rho^{h,gf}$	B	0.750	0.112	0.836	0.101	0.648	0.798	0.909
$\rho^{\xi,l}$	B	0.750	0.112	0.957	0.018	0.930	0.958	0.976
$\rho^{z,k}$	B	0.750	0.112	0.803	0.147	0.520	0.746	0.909
$\rho^{z,m}$	B	0.750	0.112	0.544	0.085	0.413	0.528	0.630
$\Gamma^{z,k}$	N	1.004	0.002	1.007	0.001	1.005	1.007	1.008
$\Gamma^{z,m}$	N	1.004	0.001	1.002	0.001	1.002	1.002	1.003
$\bar{\Pi}^{c}$	N	1.005	0.002	1.009	0.001	1.008	1.009	1.009
$\sigma_{\xi,b}$	I	3.000	2.000	9.617	4.235	5.755	9.676	15.926
$\sigma_{h,gf}$	I	1.000	2.000	0.564	0.126	0.445	0.584	0.768
$\sigma_{\xi,l}$	I	3.000	2.000	7.008	3.240	5.763	9.226	13.991
σ_{R}	I	0.200	2.000	0.111	0.011	0.104	0.117	0.132
$\sigma_{z,k}$	I	0.500	2.000	0.356	0.227	0.314	0.481	0.863
$\sigma_{z,m}$	I	0.500	2.000	0.539	0.055	0.479	0.543	0.621
$\sigma_{\theta,y,c}$	I	0.500	2.000	0.241	0.026	0.209	0.238	0.275
$\sigma_{\theta,y,k}$	I	0.500	2.000	0.304	0.034	0.273	0.312	0.359
$\sigma_{\theta,w}$	I	0.500	2.000	0.638	0.059	0.576	0.643	0.726

Figure 1: Prior and posterior distributions

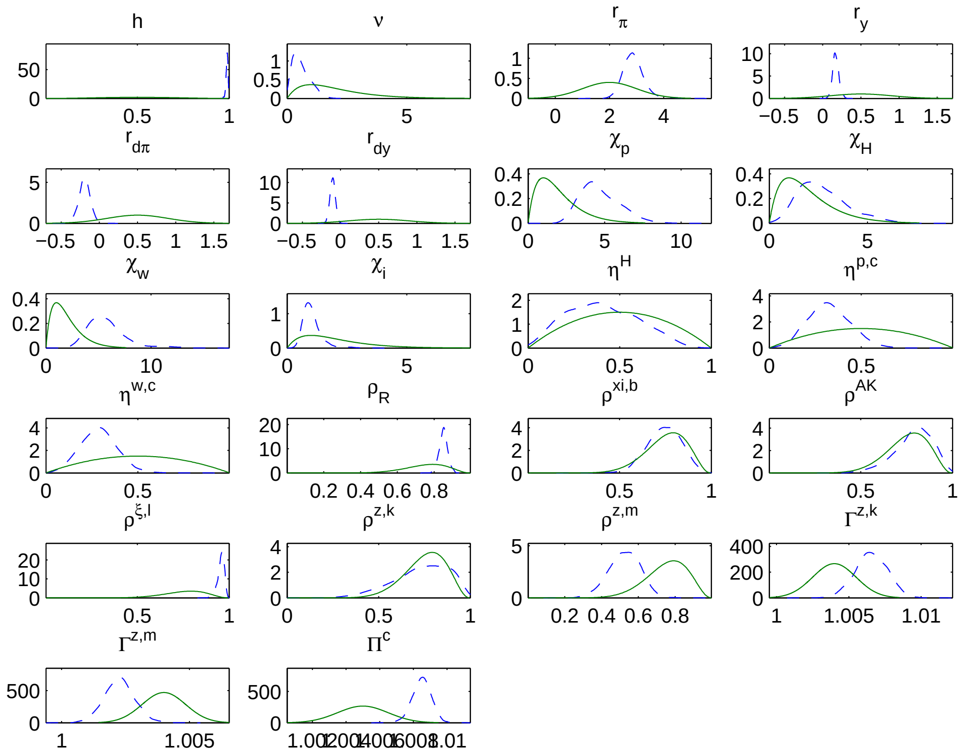

Table 2: Variance decomposition: real indicators

Shocks	Horizon	$\triangle c_t$	$\triangle i_t$	$\triangle y_t$	$\triangle w_t$	$Agg.\ Hours$
$\epsilon_t^{\xi,b}$	1	(0.83,0.91,0.95)	(0.00,0.00,0.01)	(0.02,0.03,0.04)	(0.00,0.00,0.00)	(0.05,0.07,0.09)
	5	(0.65,0.77,0.87)	(0.00,0.00,0.00)	(0.01,0.02,0.03)	(0.00,0.00,0.00)	(0.04,0.05,0.07)
	10	(0.42,0.59,0.73)	(0.00,0.00,0.00)	(0.01,0.02,0.03)	(0.00,0.00,0.00)	(0.03,0.05,0.07)
	40	(0.07,0.15,0.26)	(0.01,0.01,0.03)	(0.06,0.11,0.21)	(0.01,0.01,0.02)	(0.14,0.24,0.37)
$\epsilon_t^{h,gf}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.05,0.07,0.10)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\xi,l}$	1	(0.02,0.04,0.09)	(0.34,0.44,0.50)	(0.23,0.28,0.33)	(0.01,0.02,0.03)	(0.53,0.64,0.71)
	5	(0.05,0.10,0.18)	(0.46,0.58,0.65)	(0.39,0.48,0.55)	(0.18,0.25,0.34)	(0.67,0.77,0.82)
	10	(0.09,0.18,0.30)	(0.53,0.65,0.73)	(0.47,0.57,0.65)	(0.42,0.52,0.61)	(0.71,0.81,0.86)
	40	(0.04,0.13,0.27)	(0.40,0.67,0.79)	(0.11,0.36,0.63)	(0.70,0.80,0.86)	(0.02,0.14,0.35)
ϵ_t^{R}	1	(0.00,0.00,0.00)	(0.00,0.00,0.01)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.01,0.01)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.01)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon^{z,k}$	1	(0.00,0.01,0.02)	(0.09,0.16,0.28)	(0.03,0.06,0.12)	(0.00,0.00,0.00)	(0.03,0.06,0.13)
	5	(0.01,0.02,0.04)	(0.07,0.13,0.24)	(0.03,0.07,0.14)	(0.00,0.01,0.01)	(0.03,0.06,0.13)
	10	(0.02,0.04,0.09)	(0.06,0.12,0.25)	(0.04,0.08,0.16)	(0.01,0.02,0.04)	(0.02,0.05,0.13)
	40	(0.08,0.14,0.27)	(0.01,0.06,0.29)	(0.03,0.10,0.24)	(0.02,0.05,0.12)	(0.00,0.01,0.03)
$\epsilon^{z,m}$	1	(0.02,0.03,0.05)	(0.27,0.31,0.36)	(0.44,0.49,0.54)	(0.00,0.00,0.00)	(0.09,0.13,0.17)
	5	(0.04,0.07,0.10)	(0.17,0.21,0.26)	(0.30,0.35,0.41)	(0.01,0.02,0.04)	(0.01,0.02,0.04)
	10	(0.07,0.12,0.18)	(0.12,0.15,0.19)	(0.21,0.26,0.32)	(0.02,0.05,0.10)	(0.00,0.00,0.01)
	40	(0.26,0.39,0.53)	(0.04,0.08,0.13)	(0.04,0.14,0.33)	(0.00,0.01,0.02)	(0.29,0.48,0.63)
$\epsilon_t^{\theta,y,c}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.01,0.01)	(0.00,0.01,0.01)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\theta,y,k}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.01)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.01)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.01)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\theta,w}$	1	(0.00,0.00,0.00)	(0.03,0.04,0.06)	(0.02,0.03,0.04)	(0.97,0.98,0.99)	(0.04,0.06,0.08)
	5	(0.00,0.00,0.00)	(0.03,0.05,0.06)	(0.03,0.03,0.05)	(0.61,0.70,0.78)	(0.04,0.06,0.09)
	10	(0.00,0.00,0.00)	(0.02,0.03,0.05)	(0.02,0.03,0.04)	(0.28,0.35,0.45)	(0.03,0.05,0.07)
	40	(0.00,0.00,0.00)	(0.04,0.07,0.11)	(0.02,0.06,0.10)	(0.07,0.09,0.14)	(0.00,0.00,0.01)

Table 3: Variance decomposition: nominal indicators

Shocks	Horizon	π_t^{GDP}	π_t^c	π_t^k	R_t
$\epsilon_t^{\xi,b}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.01)
	5	(0.01,0.01,0.02)	(0.01,0.01,0.02)	(0.00,0.00,0.01)	(0.04,0.06,0.08)
	10	(0.01,0.01,0.02)	(0.01,0.01,0.02)	(0.00,0.00,0.01)	(0.08,0.12,0.17)
	40	(0.00,0.00,0.01)	(0.00,0.00,0.01)	(0.00,0.00,0.01)	(0.01,0.02,0.04)
$\epsilon_t^{h,gf}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.01,0.01)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.01,0.01)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\xi,b}$	1	(0.02,0.04,0.05)	(0.03,0.04,0.06)	(0.00,0.00,0.01)	(0.00,0.00,0.01)
	5	(0.13,0.20,0.28)	(0.16,0.24,0.31)	(0.00,0.02,0.05)	(0.01,0.05,0.12)
	10	(0.26,0.36,0.44)	(0.30,0.40,0.48)	(0.01,0.03,0.08)	(0.01,0.04,0.12)
	40	(0.61,0.71,0.78)	(0.64,0.73,0.80)	(0.32,0.48,0.59)	(0.62,0.71,0.77)
e_t^R	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.78,0.83,0.88)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.30,0.35,0.41)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.14,0.18,0.24)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.01,0.01,0.01)
$\epsilon^{z,k}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.04,0.06,0.09)	(0.00,0.01,0.01)
	5	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.37,0.48,0.58)	(0.05,0.08,0.15)
	10	(0.00,0.00,0.01)	(0.00,0.00,0.00)	(0.49,0.62,0.71)	(0.06,0.10,0.22)
	40	(0.01,0.03,0.09)	(0.01,0.02,0.05)	(0.17,0.28,0.48)	(0.00,0.01,0.04)
$\epsilon^{z,m}$	1	(0.09,0.12,0.16)	(0.08,0.10,0.14)	(0.02,0.02,0.04)	(0.00,0.01,0.01)
	5	(0.36,0.43,0.51)	(0.34,0.42,0.49)	(0.12,0.17,0.24)	(0.00,0.01,0.03)
	10	(0.34,0.41,0.49)	(0.32,0.39,0.47)	(0.11,0.18,0.25)	(0.02,0.05,0.13)
	40	(0.14,0.18,0.24)	(0.13,0.18,0.24)	(0.10,0.14,0.18)	(0.12,0.15,0.21)
$\epsilon_t^{\theta,c}$	1	(0.64,0.70,0.74)	(0.72,0.77,0.82)	(0.00,0.00,0.00)	(0.08,0.12,0.16)
	5	(0.05,0.07,0.09)	(0.05,0.08,0.11)	(0.00,0.00,0.00)	(0.20,0.25,0.30)
	10	(0.00,0.01,0.01)	(0.00,0.01,0.01)	(0.00,0.00,0.00)	(0.08,0.11,0.16)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\theta,k}$	1	(0.04,0.05,0.06)	(0.00,0.00,0.00)	(0.84,0.88,0.91)	(0.00,0.01,0.01)
	5	(0.00,0.01,0.01)	(0.00,0.00,0.00)	(0.10,0.13,0.19)	(0.00,0.01,0.01)
	10	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.01,0.01,0.02)	(0.00,0.00,0.00)
	40	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon_t^{\theta,w}$	1	(0.06,0.08,0.11)	(0.05,0.07,0.09)	(0.02,0.03,0.03)	(0.00,0.00,0.01)
	5	(0.18,0.24,0.31)	(0.16,0.22,0.28)	(0.10,0.14,0.18)	(0.05,0.09,0.12)
	10	(0.13,0.19,0.25)	(0.12,0.17,0.23)	(0.08,0.12,0.16)	(0.15,0.21,0.28)
	40	(0.03,0.05,0.07)	(0.03,0.05,0.07)	(0.03,0.05,0.07)	(0.05,0.08,0.11)

Figure 2: Impulses Responses (Two-Sector Economy)

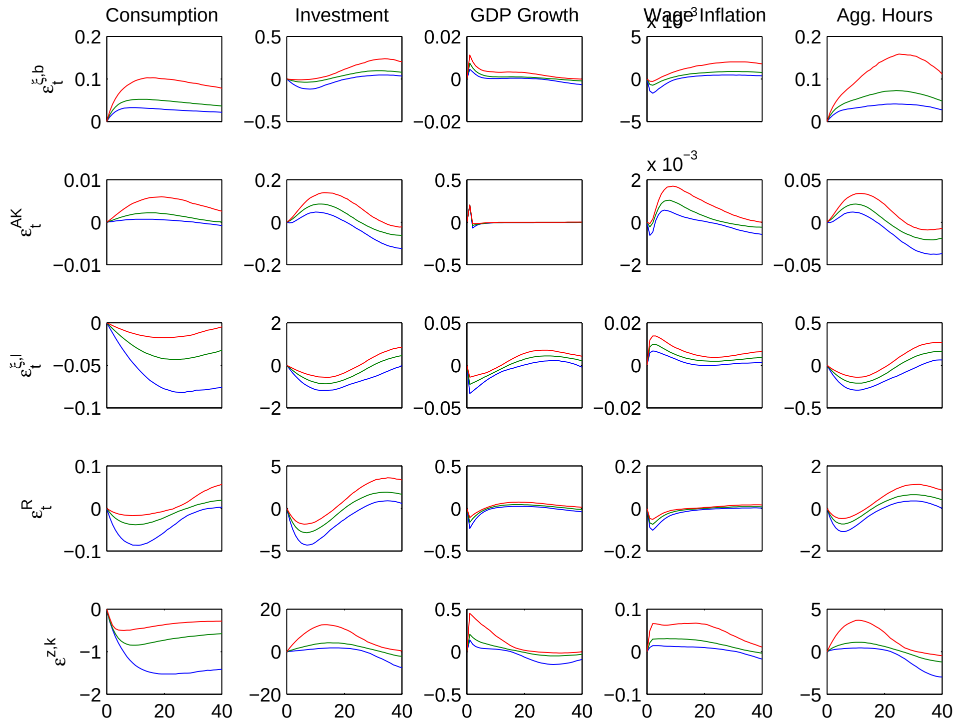

Figure 3: Impulses Responses

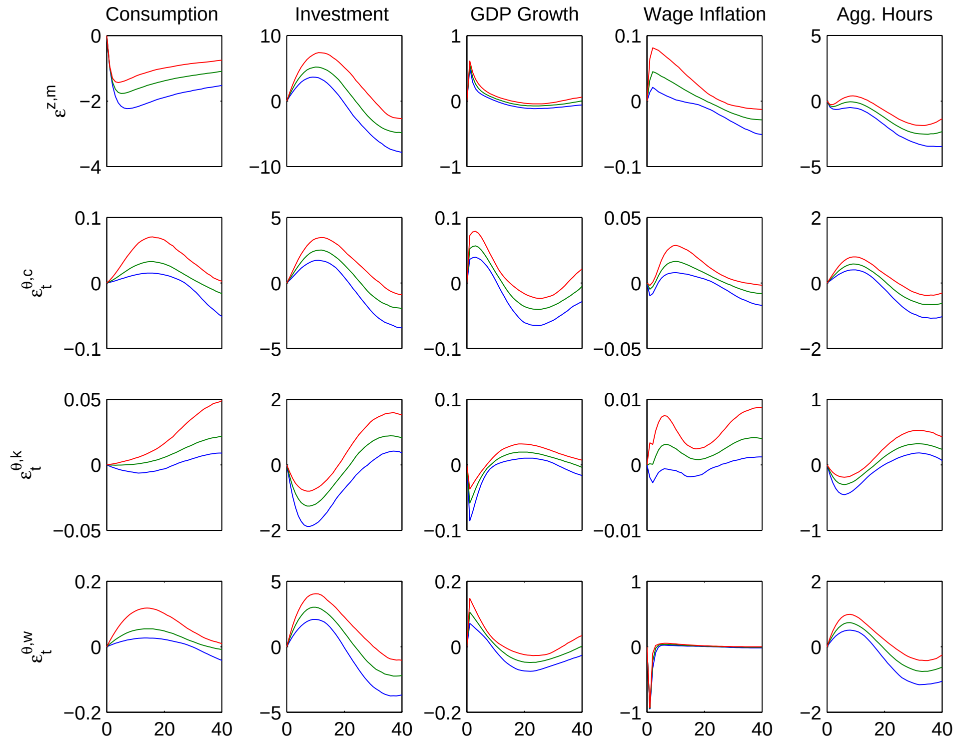

Figure 4: Impulses Responses

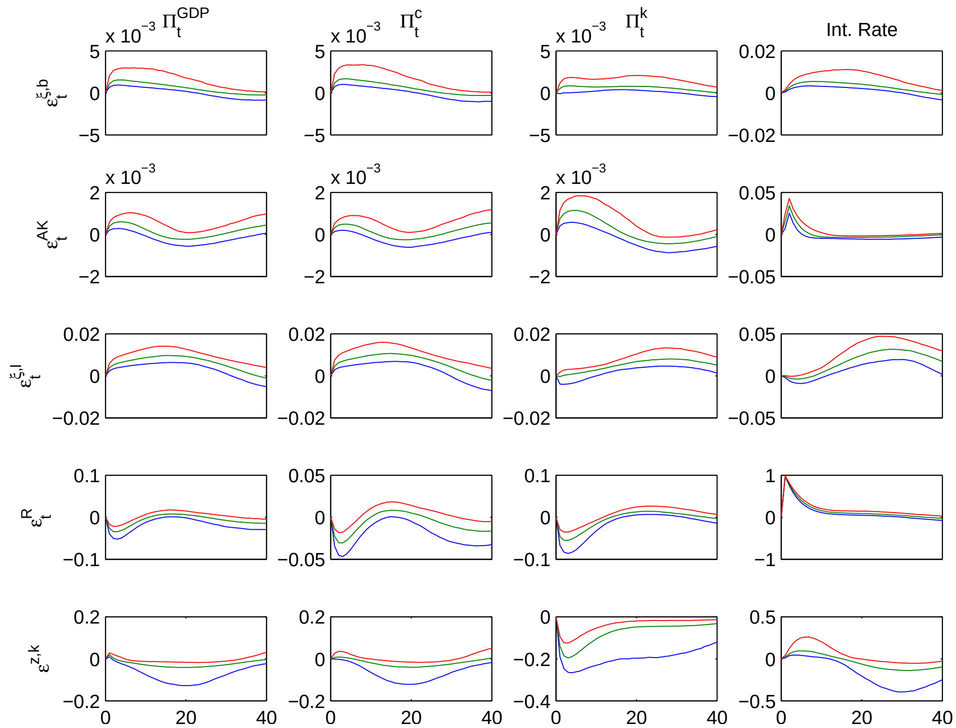

Figure 5: Impulses Responses

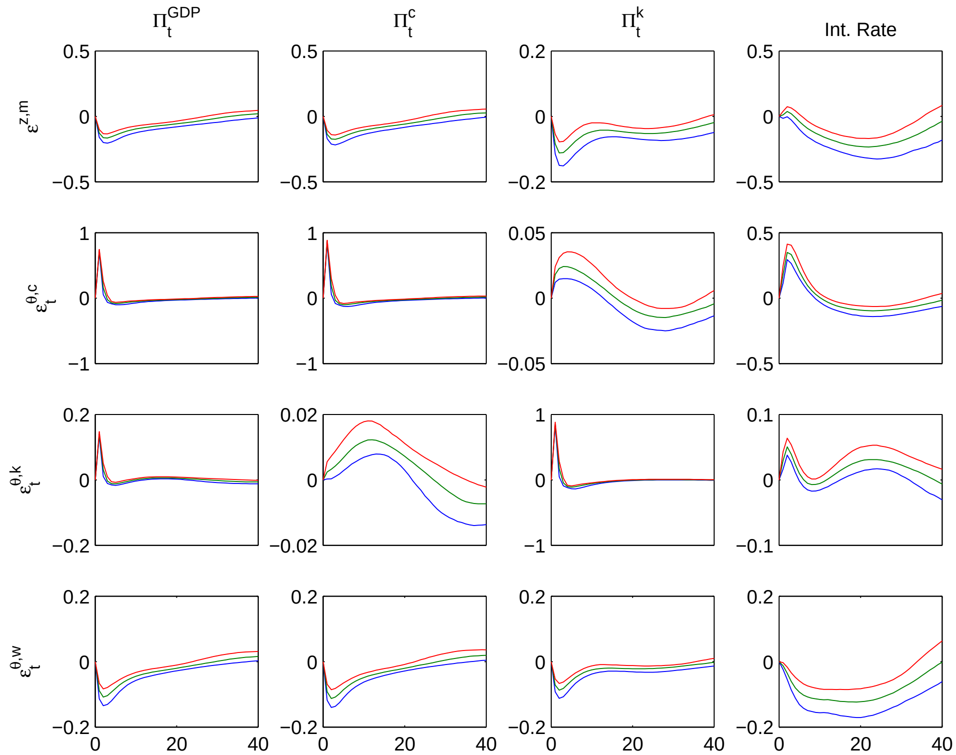

Figure 6: Prediction errors and actual data

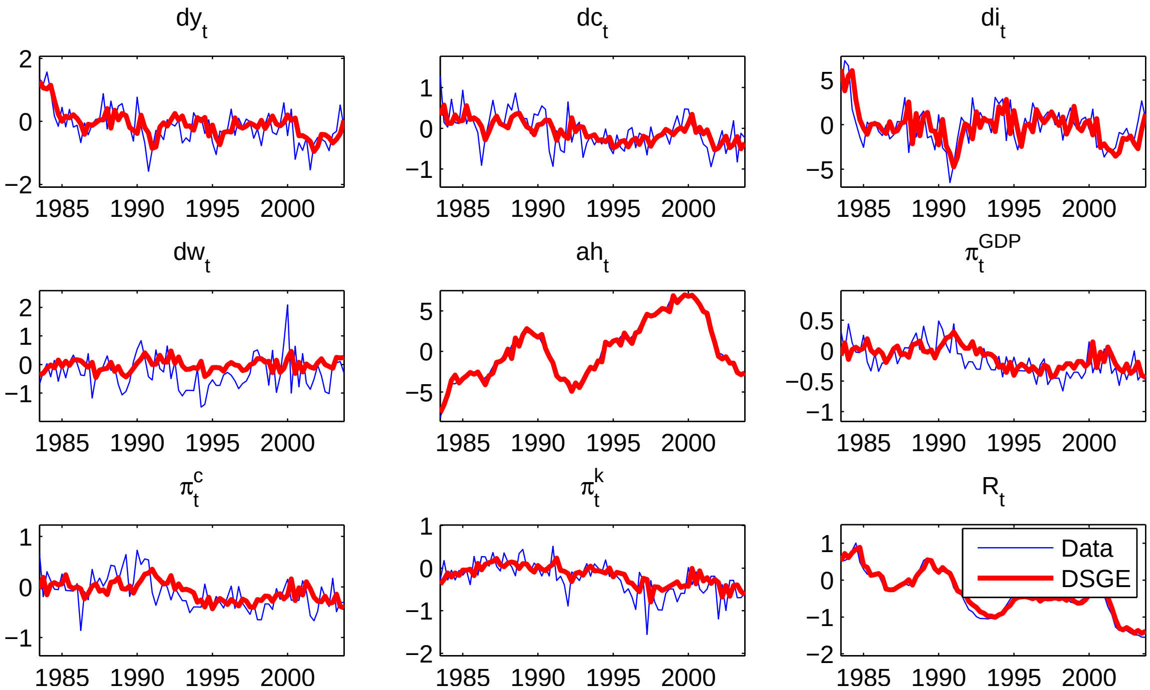

Figure 7: Paths of persistent shocks (Two-Sector Economy)

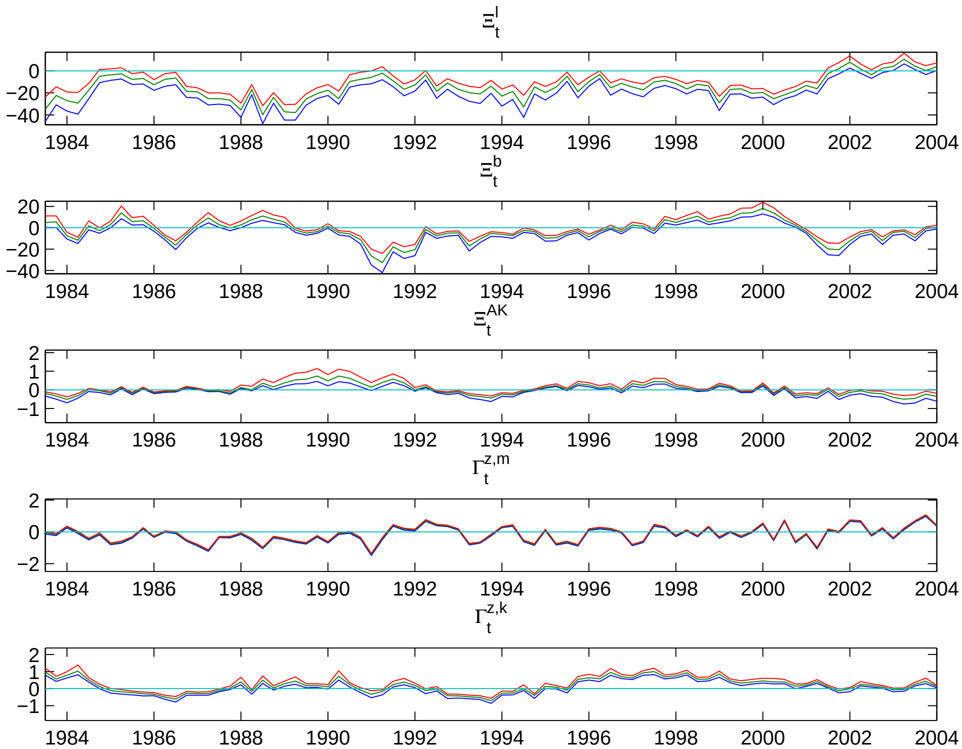

Figure 8: Output Gap Measures

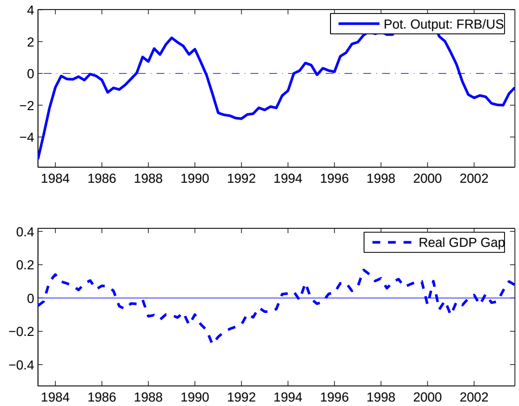

Figure 9: Realized and Natural Real Rates

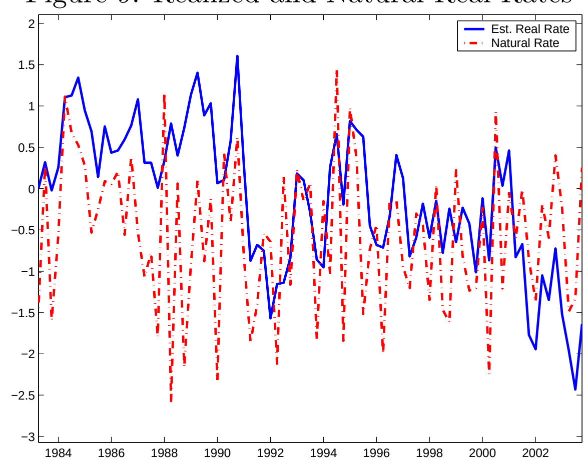

Figure 10: Real rate gap and fed funds rate paths

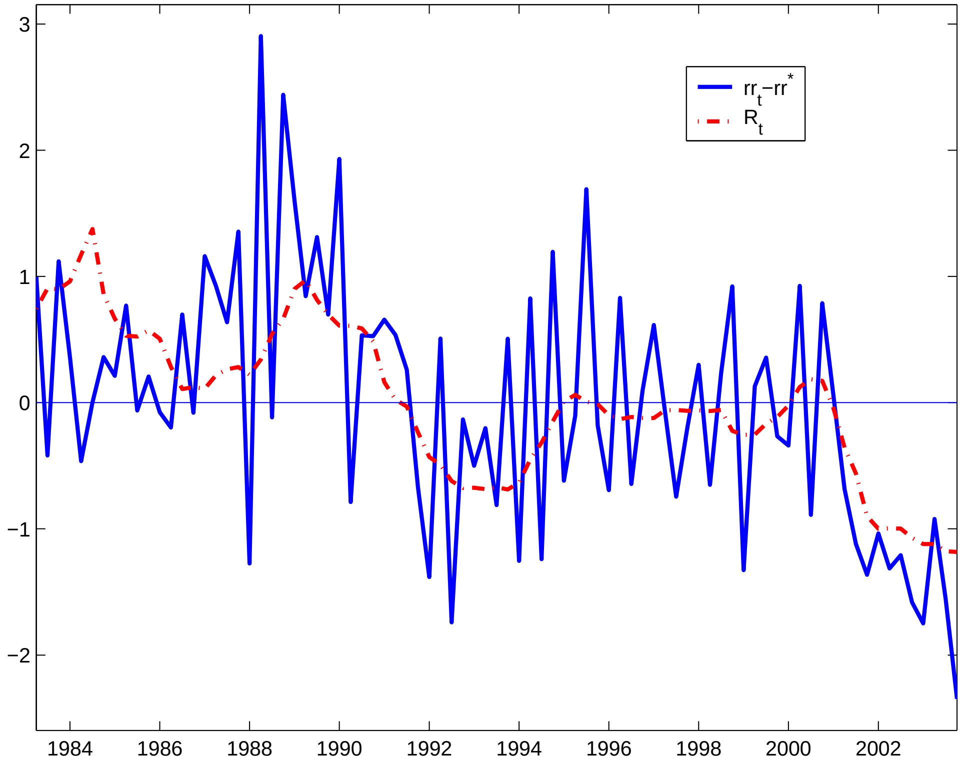

Figure 11: Welfare Surface: Observable Indicators and ρ_R=0.852.

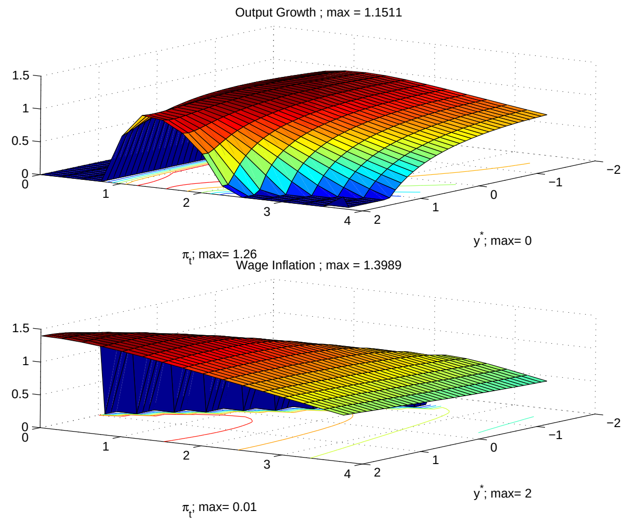

Figure 12: Welfare Surface: Unobservable Indicators and ρ_R=0.852.

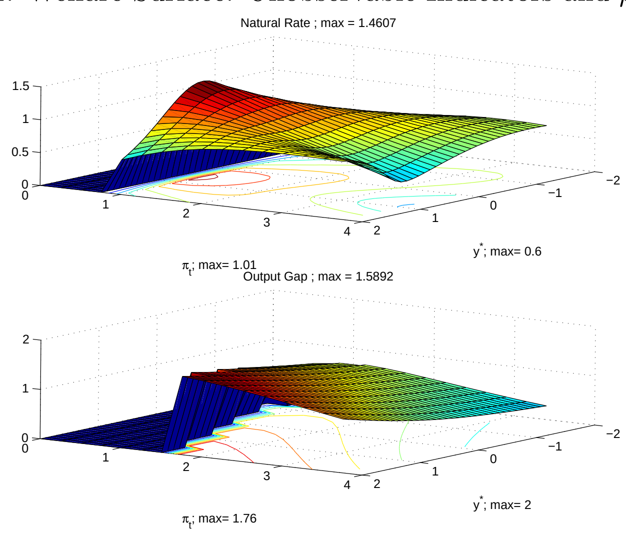

Figure 13: Welfare Surface: Observable Indicators and ρ_R=1.

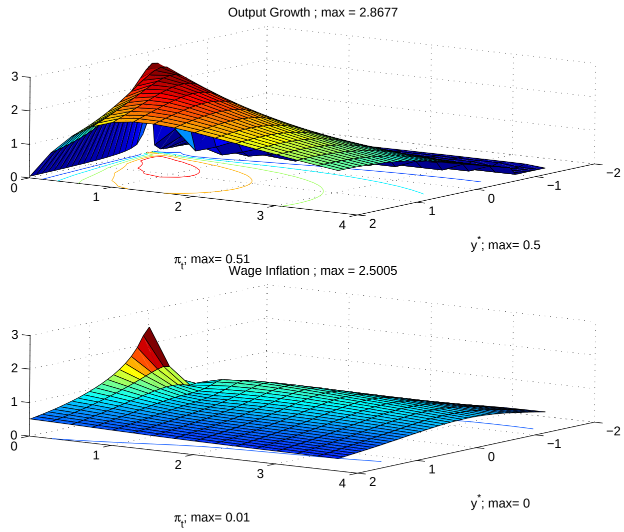

Figure 14: Welfare Surface: Unobservable Indicators and ρ_R=1.